Memristics of Second-Order Signal Theory

In memristive systems signals as carriers of information are inverting their role towards informational media.

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Abstract

Interesting new aspects are opened up for signal and information theory by the advent of memristive systems. It is proposed to map the complexity of memristive behaviors onto kenogrammatic systems with the aim to develop distributed measures of information. Event - signal - information - contexture

1. Signals of signals?

"A Memristor is a two-terminal circuit element that may fundamentally change the way information is processed in future signal processing systems." Klaus Witrisal

1.1. Signals in memristive constellations

Signals are defining memristive behaviors.
But this is only half the picture.
There is a complementary part of the picture too: Memristive behavioural structures are defining signals in nanoscale physical systems.

A *signal* has a clear definition as a physical event that is identified and separated from other events by an observer as a unit of processing and manipulation. The physical event identified as a signal is commonly understood as a classical non-quantum phenomenon, i.e. as an event which is structurally as such independent from any observer or observational context. A signal has to be detected but this detection is not itself part of its own characteristics.

With the advent of memristive systems things are changing.
Memristive events are not anymore first- or zero-order events, able to be detected and identified as identical units, identities, but occur as second-order events, i.e. as states of states of a memristive system or mechanism. Memristive events are localized in the nano-dimension of physics, and are not well understood as classical quantum states, interpreted as qubits.

A *bit* (binary digit) is the basic unit of measuring and computing information. Regardless of its physical realization, a bit is always understood as a binary event, symbolized to be either a 0 or a 1.
In quantum physics the bit gets its definition as a *qubit*, i.e. as a superposition of the two states |0> and |1> or “ket 0” and “ket 1”. A qubit is therefore not characterized as a second-order event, i.e. a state of a state.

A signal is a carrier or vehicle of information (Zeichenträger).

Signals are carrier of information and are not themselves information.
What happens if signals are structured too, and structure is indication 'informations'. Obviously a new type of signal and information appears with memristive signals.

Information might freely change its carrier matter for another carrier. But what happens if the carrier is changing too? With such situations the abstract cybernetic statement of exchangeability of the medium is losing its reason. A second-order carrier is enabling different structures of information than a first-order carrier. Hence, the exchangeability is determined and has to reflect the complexity of the matter (substrate) of the carrier.

Signals in the traditional sense are well-defined as energetic-material "objects" lacking any "cognitive" characteristics.
Second-order signals are characterized by their cognitive properties of memory and time-dependence.
Signals are the carriers of information and meaning. Semiotically they are of 0-semioticity, i.e. the material base signs.
But again, "material base signs" are not pre-given they have to be created by identification, separation and selection.

Signal theory is studying different kinds of signals and their processing. Signals are used for data transmission from a source to a sink.

Before entering into the realm of those definitions and concepts, the simple physical fact, ontologically or phenomenologically, has to be understood: signals are based on non-animated matter.

In contrast, memristive nanophysical events are not of 0-order semioticity because of their retro-grade recursivity they are of a higher order. Simple memristive events are of second-order semioticity compared to the 0-order of signals.

Hence, memristive signals, or signals conceived as memristive events, are not the "silent" carrier of information but themselves of informational character containing memory and computational properties.

Only the classical concept of dead, i.e. atemporal matter and energy, is delivering a silent carrier for information. On the base of this paradigm of matter the big step of cybernetics to define information independently of the kind of matter makes sense. It doesn't matter in which matter my morse-information is codified.

That doesn't mean that the memristive behavior of nano-dimensional matter is perceived as mind or spirit. The whole dichotomy of mind and matter becomes obsolete in this context.

Semiotically, memristive events are cognitive and volitive phenomena.
Hence, the carrier or vehicle of a sign becomes a sign itself.

What´s after Digitalism?
"Es handelt sich hier um die schwierige Arbeit der Dekonstruktion des Zeichenkörpers als Träger der Information. Es wird entschieden gegen eine physikalistische Interpretation des Signals optiert. „Signal*" war immer schon ein verstörtes Konzept der Kommunikationstheorie."*

Kaehr, Zur Verstörung des (H)ortes der Zerstörung. Fragmente einer Entstörung,
In: Kümmel, Schüttpelz (Hsg.), *Signale der Störung*, Fink Verlag 2002
http://www.vordenker.de/rk/rk_stoerung.pdf

The destiny of the carrier in general is profoundly analyzed by Horst Völz in his eminent work about a materialistic theory of signals, information, signs and society:

Signs as memory and activity
Memory: *"Das Zeichen ist dagegen eher oder sogar meist ein* Speicherzustand*, aus dem erst der Gedanke wiedergewonnen werden muss.*"

Activity: *"Die aus der sinnlichen Wahrnehmung gewonnenen* Aktionspotentiale *regen Gedächtnisstrukturen an."*
(Horst Völz, Information I, Berlin 1982, p. 332/233)

With that, a new kind of signal and information theory has to be written.

"As the researchers explained, the basis of the memristor is that the resistance of the device can be changed and be remembered, which is physically manifested by the movement of positively charged oxygen vacancies, which are dopants in a semiconducting TiO2 film. A positive bias voltage can push the vacancies away from an electrode and increase the resistance, whereas a negative bias will attract the vacancies and decrease the resistance. If left alone, the programmed state will remain as it is for at least one year."
http://www.physorg.com/news154865950.html
Self-Programming Hybrid Memristor/Transistor Circuit Could Continue Moore's Law

The nano-physical behavior of the matter, i.e. signals delivered by the movements of positively charged oxygen, is memristive.
An abstraction on this behavior is conceptualized in the model of a memristive device, the memristor.

1.2. Two challenges to signal and information theory

Signal processing in memristive systems is mainly studied under the topics of the new chances to deal with analog and digital signals and their processing.
There are two levels in signal theory where memristics might force radical changes:
1. the memristivity of the signal itself,
2. the processing modes of the signals.

How might a signal be memristive? This sounds at first absurd. A signal is a material-energetic event in space and time. But with the insight that memristivity is a phenomenon of nano-scale physics things are not such strange looking. A conceptual analysis of memristivity is discovering a chance for memristive signals too.

A first approach is considering complex bitstreams which are consisting of different types of signals which are not parallel in a isolated sense but mediated together. It turns out that just that micro-structure of mediated signal systems are ruled by memristive properties. Between different signal types, with different codes and different local sources, but both mediated, a chiasm holds. A chiasm between signal systems which is guaranteeing their heterarchical interaction is defined by pro- and retro-grade recursivity, metaphorically by "storage" and "computation", both the main properties of physical memristive systems.
In this sense, a speculative design of memristive signals as polycontextural signals is not too far from feasibility.

Considering "information processing" in biological systems, especially in neural networks, a transition from the binary concept of information to a complex memristive understanding of signals and information might be an urgent task of research.

"If memristance and STDP can be related, then
(a) recent discoveries in nanophysics and nanoelectronic principles may shed new lights into understanding the intricate molecular and physiological mechanisms behind STDP in neuroscience, and
(b) new neuromorphic-like computers built out of nanotechnology memristive devices could incorporate the biological STDP mechanisms yielding a new generation of self-adaptive ultra-high-dense intelligent machines."
Memristance can explain Spike-Time-Dependent-Plasticity in Neural Synapses
Bernabé Linares-Barranco and Teresa Serrano-Gotarredona

At the other spectre of the paradigm we have to recall Varela's "Design, reciprocity and contextuality"
"Basically the point is that in the wild an animal has to generate or define what is to be learned as these are not given as predefined lists. This is equivalent to putting the stress on the autonomy *of the living system, and to realign cognition with voluntary action rather than information processing."*

"Thus, every synaptic action is contextualized *by the pattern of activity of the constellation of inputs arrving at that point. The temptation to simplify this situation into just pre- and post-synaptic activity is great, but it surely distorts the actual neuronal dynamics far too much."*

"The substrate for these "voluntary" states is the universal reciprocity *of connections between brain regions which makes it possible for central and periphal regions to cooperate in the generation of a global state, compatible both the animal's history and the current sensory coupling."* Francesco Varela, in: Cognitiva 85, p. 762/763

The proposed sketch of a conceptual analysis or "informatic speculation" is trying to apply recent discoveries into the nano-dimension of the physical substrate of signals as they occur in memristive systems.
Polycontextural bitstreams are the carrier of polycontextural information. Hence, as a consequence of memristive bitstreams a memristive concept of infromation follows which is demanding for a memristive, i.e. polycontextural information theory.

In the following, a very simple approach to a modeling of polycontexturally distributed and mediated bitstreams is proposed.

In the chain of terms no concept is pre-given, all have to be established by thematizations and abstractions.
A signal is the energetic-material carrier of information. This abstraction is connected with the following dichotomous terms
continuous - /discrete-time
analog/digital,
signal/information,
signal/system, and
mono-/polycontextural.

From the point of view of polycontexturality, morphogrammatics and memristitive systems all mentioned dichotomies and dualityties might be involved into chiastic echangeability and simulataneity. Memristors are understood to be characterized by digital and/or analog properties. For a mono-contextural approach, distinctions are excliuive: either this or that, not both at once.
Memristive complex bitstreams might incorporate both at once: digital and analog signal

processing.

Common conceptual features of memristors, neurons and morphograms
- retro-grade recursivity, reciprocity, memrization
- locality, embodyness, contextu(r)ality,
- dynamics of cognitive and volitive actions,
- simultaneity of memory and computing functions
- transclassical, non-linear, non-dichotomic, non-atomistic,
- pattern, Gestalt, assemblies.

http://memristors.memristics.com/MorphoProgramming/Morphogrammatic%20Programming.pdf

Hence, a conceptualization which is more close to the features of nano-physical memristivity seems to be wLibyansh the risk.

1.2.1. Necessity of machine-readable signs

The general ontological and cosmological matrix of (god, world, human) of the marxist efforts to develop a materialistically founded semiotics and semantics was still dominated by humanist presumptions: it is the human being who is perceiving, reading and understanding signals, and transforming them into information, meanings and giving them a significance as signs; and nobody else

Today it is not enough to demolish the belief in transcendent forces and spirituality to achieve a materialistic solution, what was, and still is needed, is an approach or paradigm where a constellation is promoted in which machines are made able to understand materialized signs and their meaning and operational significance.

How to realize signs in technical systems?
The challenge is much more intricate than it is perceived by the theoreticians and programmer of the Semantic Web 3.0.

Computational Semiotics refers to the attempt of emulating the semiosis cycle within a digital computer.
http://www.dca.fee.unicamp.br/~gudwin/compsemio/

Semiotical computation refers to the attempt of realizing organizational chiasms of memristive operations enabling post-digital computations.

Hence, memristive systems are realizing semiotic systems, and therefore, semiotic systems are depending on material memristic activities.
It is not the dump matter or the illuminated spirit which is enabling signs and sign-using agents, but time- and history based nanophysical matter, technically realized by memristive systems, which are enabling semiosis and semioticity.

According to the eminent semiotician Alfred Toth we have to consider a fundamental change of paradigm:

"Transzendenz beginnt also im Seienden, und zwar wohl im daseinsmässigen Seienden, aber es kann sowohl im daseinsmässigen als auch im nicht-daseins-mässigen Seienden enden.

"Es liegt auf der Hand, dass die Transzendenz nach Heidegger gerade nicht-daseinsmässiges Seienden schafft. Dass dieser „Weltent-wurf" mittels Zeichen geschieht, darüber spricht Heidegger zwar nicht, aber falls es sich so verhält, dann stehen wir vor der Tatsache, dass die Transzendenz als dem Seienden innewohnender „Überstieg" die

Zeichen schafft und nicht umgekehrt die Transzendenz erst durch Zeichen geschaffen wird, wie in Toth (2010) argumentiert wurde.

"Nimmt man hingegen wie in Toth (2009) an, dass nicht die Transzendenz das Zeichen schafft, sondern dass umgekehrt die Zeichen die Transzendenz schaffen, dann ermöglicht das sowohl die Kenose zur Begründung der Semiose als es ebenfalls den Willen eines Bewusstseins, ein Objekt (dank vorausgesetzter Kenose) der Semiose zuzuführen und es also zum Zeichen zu metaobjektivieren, nicht ausschliesst." (Alfred Toth)
http://www.mathematical-semiotics.com/pdf/Z.%20als%20nicht%20daseinsmaess.%20Seiendes.pdf

1.3. Citations from Pershin and Di Ventra

Developments in the study of memristive systems are very recent. There is no surprise that there are also different approaches involved. One leading approach is elaborated by Pershin and Di Ventura.
In contrast to my own conceptual speculations Pershin and Di Ventra are working on a strictly scientific level of nanophysics and its consequences for memristive systems.
http://memristors.memristics.com/Memristors.html

Di Ventra and Pershin
"*Memory effects are ubiquitous in nature and are particularly relevant at the nanoscale where the dynamical properties of electrons and ions strongly depend on the history of the system, at least within certain time scales. We review here the memory properties of various materials and systems which appear most strikingly in their non-trivial time-dependent resistive, capacitative and inductive characteristics. We describe these characteristics within the framework of memristors, memcapacitors and meminductors, namely memory circuit elements whose properties depend on the history and state of the system.*"

"*memory* is the ability to store the state of a system at a given time, and access such information at a later time."
http://arXiv:1011.3053

2. From Signals to Information

If memristive signals are of second-order then it follows that memristive information is at least of second-order too. Hence, a memristive information theory has to be developed on the base of the new situation for a definition of information.

Memristive systems are described as having a state and a memory of this state. In several papers I tried to explain the second-order character of memristive behaviors. Even without reflectional thematizations it should be clear that the memristivity of a memristor is defined by 3 relations (features, contexts, qualities):
1. the resistance of the actual behavior of the memristor as a resistor,
2. the memory of the state of the past behavior of the memristor as a storage, and
3. the relationship between the state of the first and the state of the state, meta-state, of the second behavior, memristor as computation.

Information is measured by bits. Such binaries are easily distributed over different carriers of different complexity. Therefore, a second-order signal, carrying a second-order information is measured by a mediation of first- and second-order measures, i.e. by a triple consisting of two binaries and one mediating binary.
Instead of bits, with 0 and 1, we get a simple complexion of a triple of bits: (0-1, 0-1, 0-1) as

$(1_{1.3} - 1_1/0_2 - 1_{2.3})$.

Distributed binary digits – bits :

$$\text{bit}^{(3.1)} = \left(\text{bit}^{1.1}, \text{bit}^{2.2}, \text{bit}^{3.3}\right) = \begin{pmatrix} 0 & \square & \square \\ \square & 1 \cong 0 & \square \\ \square & \square & 1 \end{pmatrix} = \begin{pmatrix} \text{bit}^{1.1} & \square & \square \\ \square & \text{bit}^{2.2} & \square \\ \square & \square & \text{bit}^{3.3} \end{pmatrix}$$

$$\text{bit}^{(3.1)} = \boxed{\begin{array}{ccc} 0 & \longrightarrow & 1 \cong 0 \\ & \searrow & \downarrow \\ & & 1 \end{array}}$$

Mediated binary systems have certainly to realize the *matching conditions* of their base systems, hence

$\text{MC}(\text{bit}^{(3.1)}) = (0_1 \cong 0_3, 1_2 \cong 1_3, 0_2 \cong 1_1)$

Second-order objects, like 3-contextural binary digits ($\text{bit}^{(3.1)}$), are defined by 3 sub-systems, the first- and second-order relational object (sub-system) and the third super-additive mediational sub-system between both sub-systems.

3 – binary arithmetics

$$N^{(1,1)} = \boxed{\begin{array}{ccc} 0 & \longrightarrow & N \\ & \searrow & \downarrow \\ & & N \end{array}}$$

$$N_{par}^{(3,1)} = \boxed{\begin{array}{ccc} 0 & \longrightarrow & N \\ & \searrow & \downarrow \\ & & N \end{array}} \amalg_{1.2} \boxed{\begin{array}{ccc} 1 & \longrightarrow & N \\ & \searrow & \downarrow \\ & & N \end{array}} \amalg_{1.2.3} \boxed{\begin{array}{ccc} 0 & \longrightarrow & N \\ & \searrow & \downarrow \\ & & N \end{array}}$$

Null

$\mathbf{N}_{par}^{(3,1)}$:

$$\left[\begin{array}{c} \left(\begin{array}{c} (0 \longrightarrow N)_{1.0.0} \\ \amalg_{1.2} \\ (1 \longrightarrow N)_{0.2.0} \end{array}\right) \\ \amalg_{1.2.3} \\ (0 \longrightarrow N)_{0.0.3} \end{array}\right] \circ_{1.2.3} \left[\begin{array}{c} \left(\begin{array}{c} (N \longrightarrow N)_{1.0.0} \\ \amalg_{1.2} \\ (N \longrightarrow N)_{0.2.0} \end{array}\right) \\ \amalg_{1.2.3} \\ (N \longrightarrow N)_{0.0.3} \end{array}\right] = \left[\begin{array}{c} \left(\begin{array}{c} (0 \longrightarrow N) \circ_{1.0.0} (N \longrightarrow N) \\ \amalg_{1.2} \\ (1 \longrightarrow N) \circ_{0.2.0} (N \longrightarrow N) \end{array}\right) \\ \amalg_{1.2.3} \\ (0 \longrightarrow N) \circ_{0.0.3} (N \longrightarrow N) \end{array}\right]$$

Gunther’s kenogrammatic arguments

"Thus the conclusion is inevitable that we need for the design of decision-making machines an enlarged information theory *which would have at least two measuring units to count the different kinds of information:*
*a) the conventional unit derived from Table I [*binary tree*], and*
b) a second unit derived from trito-structure *which would measure the flow of such information that is, for the present, beyond the scope of the measuring techniques of Shannon’s theory.*
This second unit would have to deal with the problem of meaning, which for the time being,

is expressly excluded from all exact theories about information." (Gotthard Gunther, Decision Making Machines, 1970, p.10)

"Considering the problem of meaning, it is quite possible that the introduction of a second unit would only be a first step in a new direction, and that would be forced to introduce an ever-increasing hierarchy of information units. This, at any rate, would be the case if we assume that our ultimate ambition is - logically speaking - to measure degrees of complexity." (ibd., p.12)

Gunther's approach might be understood as based on the distinction of *iterative* and *accretive* kenogrammatic continuations.

This is, in a first step, translated into the general polycontextural distinction of *election* and *selection*. In Gunther's terms, the distinction is between intra- and discontextueral event. And this is exactly ruled by the operators of election and selection.
http://memristors.memristics.com/MorphoReflection/Morphogrammatics%20of%20Reflection.html

Interpretation: Information, meaning and significance
Gunther is distinguishing between information and meaning. But more interestingly, he is also introducing the term "significance" into logic and information theory.
Hence, an information might be, independent of its information value, of significance in one context or contexture and of no or another significance in another context. With this change the event still keeps its properties as information, albeit context-depending meaning. But information in one context or domain might be marked with {0, 1} and in the other context with {1, 2}. Both are defining for themselves full-fledged informational domains.

The question still open is, how are Gunther's information units related?

With the insight into memristive systems the question about a possible *technical* realization of poly-information has changed dramatically.

3. Memristive Information

3.1. First-order thematizations

Memristor:
analog/digital
memory/computing

"A Memristor is a two-terminal circuit element that may fundamentally change the way information is processed in future signal *processing systems. The Memristor can be understood as a resistor whose resistance can be programmed. It therefore combines a memory element that can be programmed with a conventional resistor. How can we use this for* signal *processing? The memory can be used to store information,* while *the resistor performs multiplication and division operations - just consider the simple equation of Ohm's law."* Klaus Witrisal
http://www.spsc.tugraz.at/courses/projects-theses/Witrisal_The1

The question is, how are both features, the memory and the programming element, interacting in the new paradigm of signal processing?

On the base of a possible second-order or in general, a polycontextural information theory, the behavior of memristive systems might be more fundamentally studied.

Technically, the situation boils down to the question, how can we map binary trees onto the

trito-structure of kenogrammatics?
The proto- and deuter-structure has the nice property of commutativity which allows the statements that the way down is not necessarily the same as the way up. But the graph of the trito-structure as such is not yet offering such an easy solution.
http://www.thinkartlab.com/pkl/media/SKIZZE-0.9.5-TEIL%20A-ARCHIV.pdf

Polycontexturally distributed bitstreams are not based on “multicarrier"-concepts but on mediation.

For a first-order conceptualization of memristive signal processing:

Blaise Mouttet
"By configuring a single crosspoint to a low resistance state 16 different possible signal outputs are possible, however by allowing for the configuration of multiple crosspoints a maximum of 2^{16} = 65,536 possible signal transformations are possible [...] By using a periodic pulse as the input signal and providing a larger crossbar array with finer time delay between adjacent columns of the crossbar and a larger range of resistances for the different rows there is potential to create a universal waveform generator capable of adapting the amplitude and timing of signals in accordance with a variety of desired applications.”

"However, for large crossbar arrays this method can be time consuming. For example, a moderately small 100x100 binary crossbar array having 100 input wires and 100 output wires a total of 2^{10000} possible states (approx. equivalent to a number represented by 10^{3000} which is 1 followed by 3000 zeros). Programming based on genetic algorithms may be used to more quickly optimize the binary states of the crossbar array.”
http://knol.google.com/k/programmable-electronics-using-memristor-crossbars#SIGNAL_PROCESSING_WITH_MEMRISTORS

Klaus Witrisal
"This paper shows that such a stored-reference receiver can be used for multicarrier signals as well. Multiple output samples are acquired and a post-processing step is introduced to separate the subcarrier signals, while the pre-processing in the memristor receiver (MRX) collects energy from resolvable multipath components.
Hence a potential complexity reduction is achieved because frequency diversity is exploited without error correction decoding and without processing UWB signals at Nyquist rate. Performance simulations validate the concept of the multicarrier receiver and demonstrate the promising features of the MRX.
However, a critical comparison to a single-carrier scheme indicates that the final gain may not justify the extra complexity.”
Klaus Witrisal, A Memristor-Based Multicarrier UWB Receiver
http://www.spsc.tugraz.at/people/klaus/WitrisalICUWB09.pdf

Sandro Carrara, Fabrication of Memristors with Poly-Crystalline Silicon Nanowires Nanowires, 2009
http://si2.epfl.ch/~scarrara/Genova_July_2009_Memristors.pdf

3.2. Second-order reduction strategies

3.2.1. Contextural decomposition of big numbers

Proposed second-order concepts of signal processing with polycontextural bitstreams are candidates for *reduction strategies* of ‘astronomical' signal occurrences towards technically feasible solutions.

Reduction strategies had been proposed since the advent of polycontextural logics (G. Gunther 1962).
The basic concept of such logics is mediation. With the difficult situation that a value in one logic might be the value “true” and simultaneously in its mediated neighbor logic the value “false”. Conceptually, this problem is solved with the strategy of chiastic interchangeability constructs. It is stipulated that this mechanism gets now a technical solution within memristive systems as being able to mediate ‘memory'- and ‘computing'-functions at once.
Based on memristive mediation, new possibilities of distributing and mediating natural number series to avoid ‘astronomical’ and not-feasible quantities are getting accessible to formalization, implementation and realization.

Example

$$NN^{(m)} \xrightarrow{\text{parallelization}} \begin{pmatrix} N^1 & \longrightarrow & N^1 \\ N^2 & \longrightarrow & N^2 \\ & \cdots & \\ N^m & \longrightarrow & N^m \end{pmatrix} \xrightarrow{\text{mediation}} NN^{(m)} = \left(NN^1 \amalg NN^2 \amalg \ldots \amalg NN^m\right).$$

Dissemination of NN $^{(m)}$:

$$\left[\begin{pmatrix} \left(\mathbf{N}_{1.0.0} \quad \circ_{1.0.0} \quad m_{1.0.0}\right) \\ \amalg_{1.2} \\ \left(\mathbf{N}_{0.2.0} \quad \circ_{0.2.0} \quad m_{0.2.0}\right) \end{pmatrix} \\ \amalg_{1.2.3} \\ \left(N_{0.0.3} \quad \circ_{0.0.3} \quad m_{0.0.3}\right)\right] = \left[\begin{pmatrix} \mathbf{N}_{1.0.0} \\ \amalg_{1.2} \\ \mathbf{N}_{0.2.0} \end{pmatrix} \\ \amalg_{1.2.3} \\ \mathbf{N}_{0.0.3}\right] \circ_{1.2.3} \left[\begin{pmatrix} m_{1.0.0} \\ \amalg_{1.2} \\ m_{0.2.0} \end{pmatrix} \\ \amalg_{1.2.3} \\ m_{0.0.3}\right]$$

Dissemination of 2 ^{10000}

$$2^{10000} \xrightarrow{\text{parallelization}} \begin{pmatrix} N^1 & \longrightarrow & N^1 : 2^{1000} \\ N^2 & \longrightarrow & N^2 : 2^{1000} \\ & \cdots & \\ N^{10} & \longrightarrow & N^{10} : 2^{1000} \end{pmatrix} \xrightarrow{\text{mediation}}$$

$$\left(2^{1000} \amalg 2^{1000} \amalg \ldots \amalg 2^{1000}\right) = 2^{10000}.$$

A reasonable approach to realize parallelism is a) to ‘colour’ intracontexturally crossbar system into partitions or b) to distribute partitions over discontextural loci of poly-crossbar systems.

Existing implementations and simulations are not taking into consideration the second-order qualities of memristive systems.
http://www.thinkartlab.com/pkl/lola/FIBONACCI.pdf

3.2.2. Broken paths, gaps and jumps

Beyond the purely conceptual design of nano-electronic devices, there are *practical* obstacles towards arbitrary dimensions too. As much as there is no need to deal with astronomical or ultra-astronomical numbers, there is also no need to deal with arbitrary long distances of signal processing.

Usually there are at least two main strategies to deal with such technically unavoidable defective situations. One is to overcome the deficiencies of the physical constellation with the help of clever formal methods which allow to escape the worst drawbacks. The other strategy is to adapt the applied formalism and conceptual approaches to the imperfect realities.

An interesting and formally very interesting approach is proposed by Wenjing Rao et al.
This approach might be seen as a step of deliberating formal modeling towards a more flexible conceptualization from the matrix- to a bi-partite graph-approach.

"It can be seen from this example that mapping a variable or a product term onto a specific nanowire imposes constraints on the subsequent mapping. Therefore, (i) determining whether a logic function can be successfully mapped to a specific crossbar with topology constraints and (ii) finding a mapping, if one exists, constitute important new challenges for logic synthesis on nanelectronic crossbars." (Wenjing Rao)

Steps on such a journey of deliberating formal constraints are marked by
a) set-thereotical *matrix*-approaches, MxM --> M,
b) *relational*, B(N, N', E),
c) *functorial*, (B_1, B_2, E, f) with bi-partite graph embedding, and
d) *bifunctorial*, (($B^1 \coprod B^2$), ($E^1 \coprod E^2$), approaches.

The second strategy of dealing or interacting with changing complexity has not yet developed enough recognition to be accepted as a reasonable candidate.
The reason is simple, the second strategy, which is based on polycontextural logic and morphogrammatics, starts from the intuition of a theory of *living systems*, and still has a long way to concretize towards technical, i.e. artificial living systems.

From the point of view of a theory of living systems, path or journeys in nanoscale configurations are not pre-given but build (co-created) - *"because it is the* act of crossing *that creates the junction*", Kuekes, Philip J. -locally by interaction. Hence, the mapping of B^1 and B^2 is understood traditionally as a yuxtaposition, and polyontexturally as an interaction between (B_1, B_2, E, f) and (B_1, B_2, E, f) realized formally as a chiasm of the bifunctoriality, i.e. interchangeability and further as metamorphosis, of its constituents.

Metaphors and strategies
Some metaphors, concepts and strategic speculations to deal with gaps, jumps, simultaneities, are sketched in the next paragraph, which recalls some patterns of interactions between binary number systems and kenogrammatics.

Both approaches are in some sense opposite and probably complementary.

Citations
From a logico-arithmetical point of view of polycontexturality, Gunther emphasizes the necessity of *redundancy* in the tissue of living systems and their ‘physical’ imperfection.

"Aber da, wie bekannt, organische Systeme mit ganz erheblichen Redundanzen arbeiten, ist es wichtig, auf diese Unterschiede hinzuweisen." (Gotthard Gunther, Natürliche Zahl und Dialektik)

Challenges of design
"With nanoelectronic technologies evolving at a rapid clip towards practical realization of nanoelectronic computing systems, it is important to identify and propose research on the new challenges emerging during their design process. This paper identifies a *new challenge of mapping* a logic function onto a nanowire crossbar under the inherent

constraints of limited connectivity and unreliability. This paper sets up an important initial contribution in the logic synthesis for nanoelectronics.” (Wenjing Rao et al, p. 726)

Limited distance

"First, it is extremely difficult to transfer signals over a long distance due to *signal attenuation* in nanowires. This strictly limits the length and connectivity of nanowires.”
However, in a nanoelectronic crossbar with breaks in wires (either periodic or arbitrary), this mapping has to obey certain constraints. In the first case, the nanowires are broken periodically and the pattern of a crossbar is available at the fabric time.
In the second case, the breaks are introduced by the manufacturing defects and are rather arbitrary, thus demanding a post-fabric testing process to identify the crossbar structure pattern.”

"This paper identifies a new challenge of mapping a logic function onto a nanowire crossbar under the inherent constraints of limited connectivity and unreliability.” (Wenjing Rao et al)

Defects

Wenjing Rao , Alex Orailoglu, Ramesh Karri
Topology Aware Mapping of Logic Functions onto Nanowire-based Crossbar Architectures (2006, 723--726), IEEEACM Design Automation Conference DAC
Runtime Analysis for Defect-tolerant Logic Mapping on Nanoscale Crossbar Architectures
Yehua Su and Wenjing Rao
"Abstract--Crossbar architectures are promising in the emerging nanoscale electronic environment. Logic mapping onto highly defective crossbars emerges as a fundamental challenge and *defect tolerance* techniques therefore become crucially important.”
http://www.ece.uic.edu/~wenjing/pub/conf/2009nanoarch.pdf

Redundancy

"Instead, increasing the area of the crossbar provides enough *redundancy* to implement circuits in spite of the defects. We identify reliability thresholds in the ability of defective crossbars to implement boolean logic. These thresholds vary among different implementations of the same logical formula, allowing molecular circuit designers to trade-off reliability, circuit area, crossbar geometry and the computational complexity of locating functional components. We illustrate these choices for binary adders."
Tad Hogg and Greg Snider, Defect-tolerant Logic with Nanoscale Crossbar Circuits
http://www.springerlink.com/content/e61323p9k2px5l78/

Crossbars

"Crossbars have been proposed as an architectural approach to nano-electronic computation because of their simplicity of fabrication and inherent redundancy, which supports *defect tolerance*.”
G. Snider, Computing with hysteretic resistor crossbars, Appl. Phys. A 80, 1165-1172 (2005)

Creation

"As used herein, the term "self-aligned" as applied to "junction" means that the junction that forms the switch and/or other electrical connection between two wires is created wherever two wires, either of which may be coated or functionalized, cross each other, because it is the *act of crossing* that creates the junction."
Kuekes, Philip J. et al., Molecular-wire crossbar interconnect (MWCI) for signal routing and communications
United States Patent 6314019, 2001
http://www.freepatentsonline.com/6314019.html

Get ready for the cyberwar

An early application of polycontextural logic based on logical fibering can be found in the dissertation: Marvin Oliver Schneider, Logical Fibering Based Web Access Management, 2007. This approach is not yet taking the interchangeability concept of polycontextural monoidal categories into account. And obviously, memristive intensions are not yet visible.

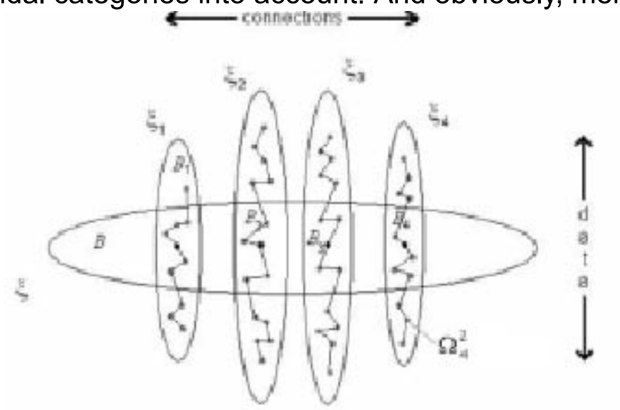


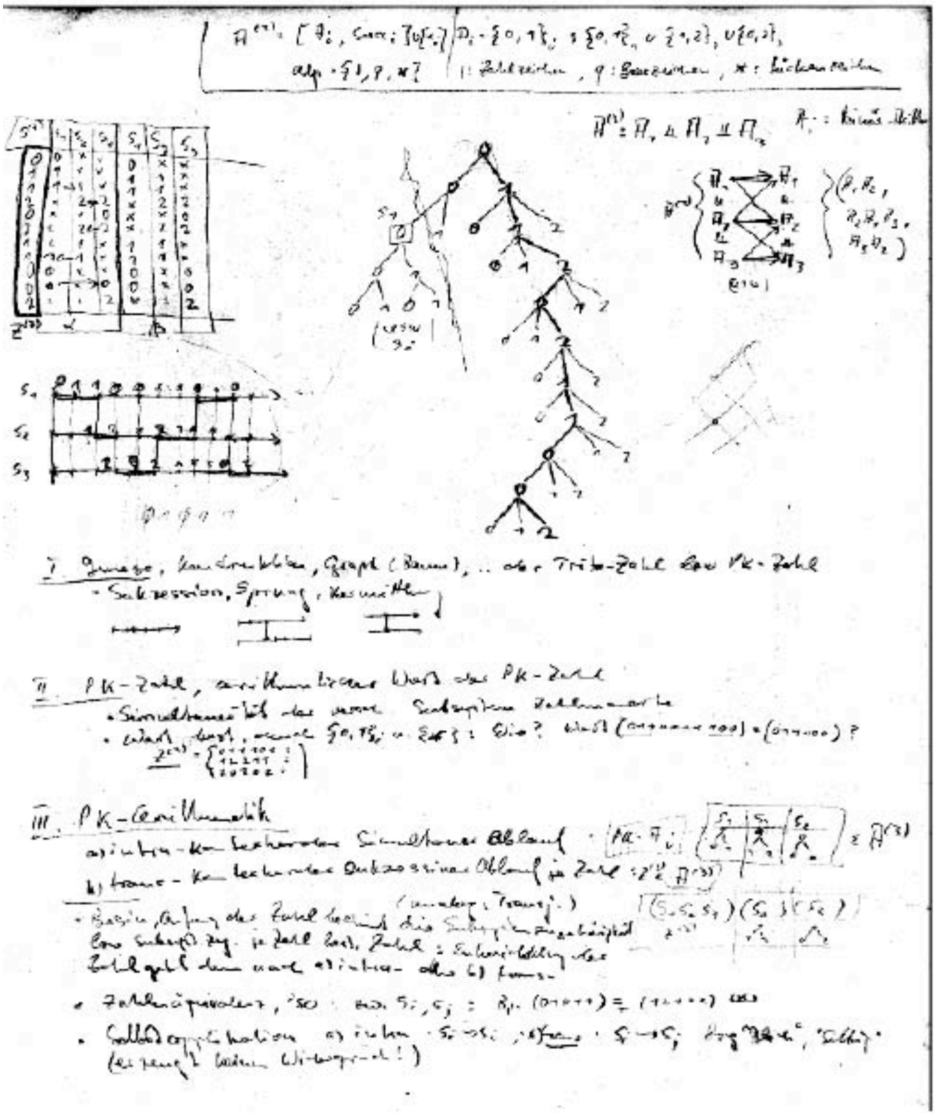
Figure 3-7: Interconnected condensed fibering (secondary data representation)

"It is interesting to note, that by its principles, a Logical Fibering is not limited concerning enumeration (see above) nor by the nature of its logics. This freedom of representation can lead to the expression of complex logical conditions. The Fibering in this case is the framework, the advanced data type, which is used.
By its native functions it may also model communication, which is, however, not used in FG [Fiber Guard], therefore bringing the need of the implementation of specific functions, which work over the Fibering as the storage and analysis algorithms shown above and the retrieval method, to bediscussed in the following part." (Schneider, p. 59)
http://digbib.ubka.uni-karlsruhe.de/volltexte/documents/4452

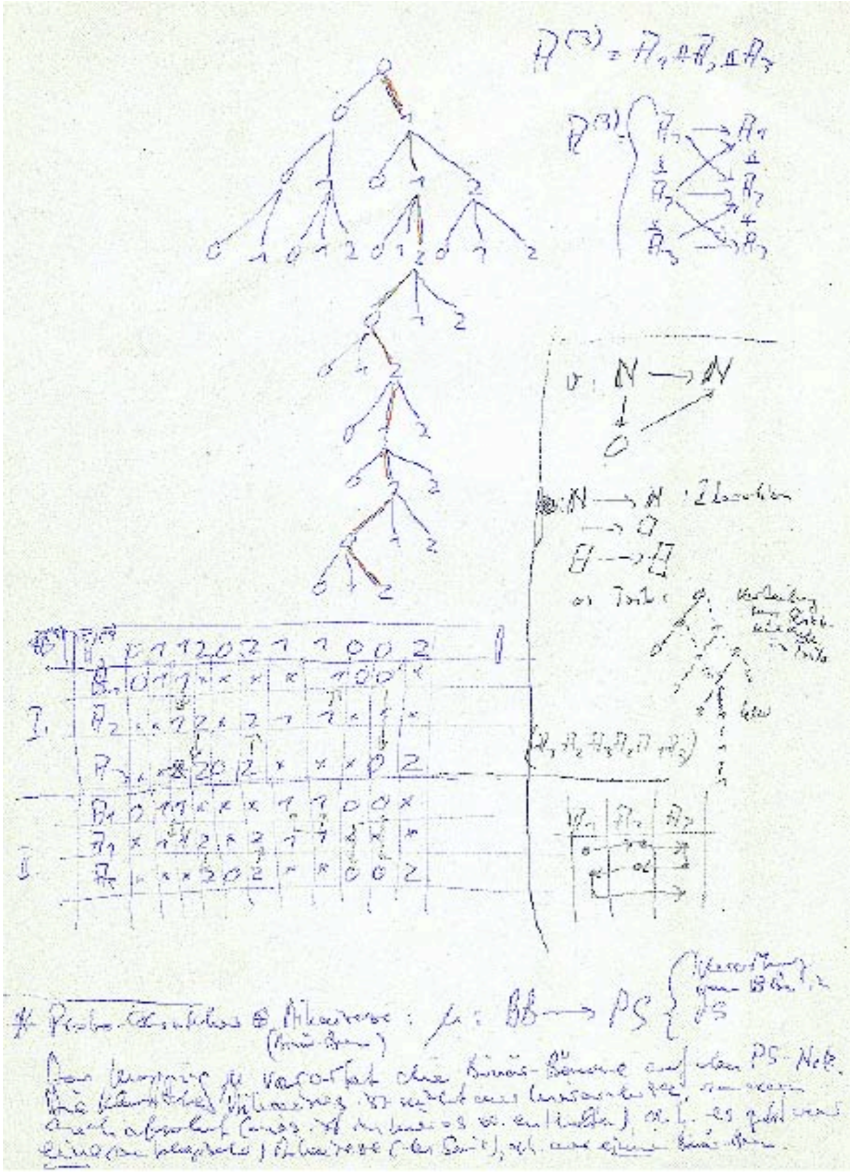
4. Polycontextural Modeling of Memristive Information

4.1. Poly-Arithmetic diagrams for binary systems

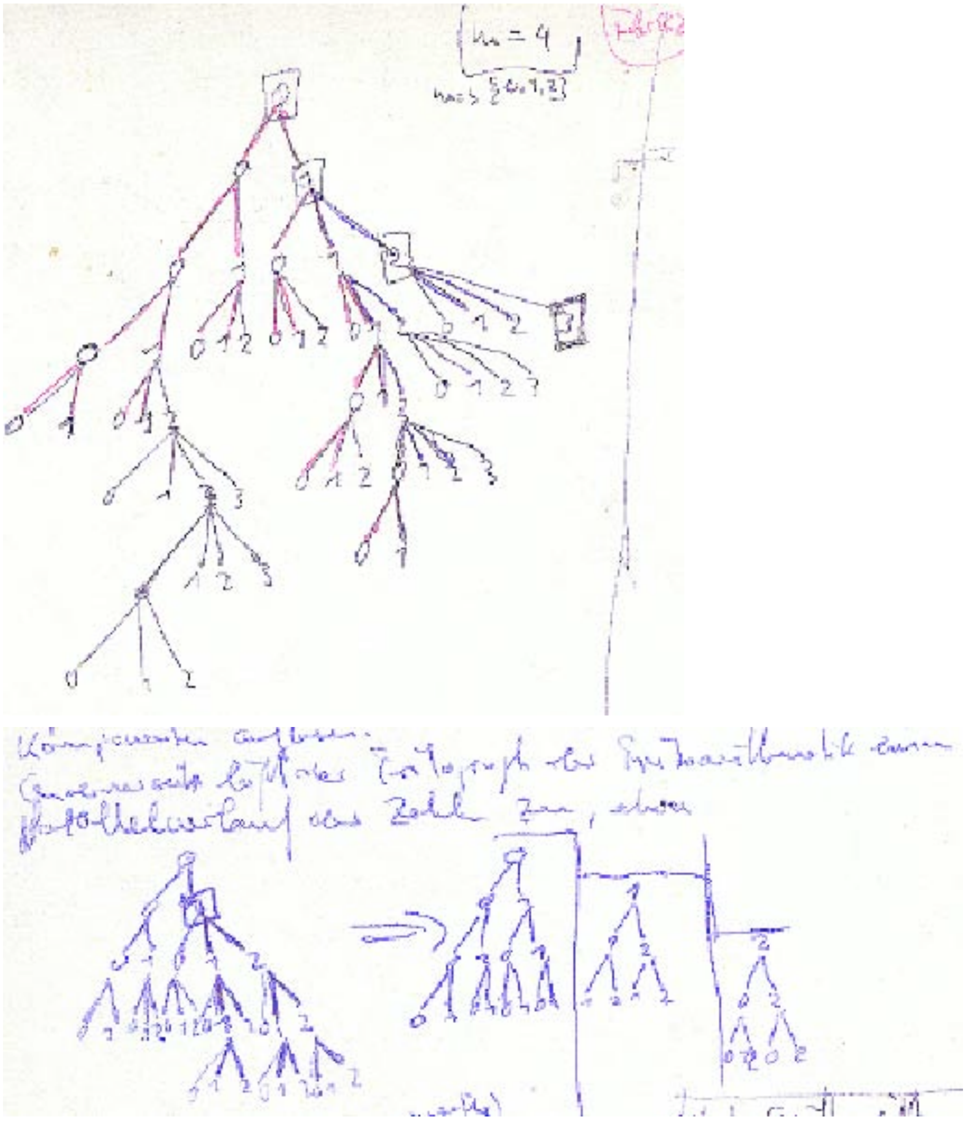
4.1.1. General scheme of disseminated binary systems



4.1.2. Decomposition of tritograms



4.1.3. Simultaneous (parallel) binary arithmetics



4.1.4. Definition of the trito-structure

Hence, following *Morphogrammatik*, p.70, we write for the *degree of accretion*, *AG*, for a kenogr
N_{TTS}:

$n_{TTS}(ts) = AG(ts) + 1$ for the set of the $n_{TTS}(ts)$
of trito kenogrammatic successor functions of a kenogrammatic sequence *ts*.

AG is : $1 \leq AG(ks) \geq |ks|$, with $|ks|$ as the *number* of kenoms in a trito – grammatic sequence

Trito – trans – successor for kenograms

$$n_{TTS}(MG) = AG(MG) + 1$$

```
fun TTS ts = map (fn i ⟹ ts @ i])
                 (fromto 1 ((AG ts) + 1));
```

Example

$MG = [1, 2, 3]$

$AG(MG) = AG([1, 2, 3] = 3$

$n_{TTS\,1}(MG) = [1, 2, 3, 1]$

$n_{TTS_2}(MG) = [1, 2, 3, 2]$

$n_{TTS_3}(MG) = [1, 2, 3, 3]$

$n_{TTS_4}(MG) = [1, 2, 3, 4], AG(MG) + 1$

Recursive TTS

$MG_0 \in TTS$

$MG \in TTS \implies n_{TTS}(MG) \in TTS$

Cardinality

The cardinality of trito-sequences TTS is calculated by the sum of Stirling numbers of the second kind $\sum_{k=0}^{n} S(n, k)$, i.e. with S(n, k) = S(n - 1, k - 1) + k * S(n - 1, k).

http://mathworld.wolfram.com/StirlingNumberoftheSecondKind.html

4.2. Poly-Bitstreams

4.2.1. Parallel bitstreams

Poly – bitstreams are binary interpretations of trito – kenogrammatic structures TTS.

parallel bitstream $A^{(3)}$ =

S1	0	**1**	1	1	1	–
S2	–	**1**	**2**	1	1	–
S3	–	–	**2**	2	2	0
time	t1	t2	t3	t4	t5	t6

Parallel bitstreams may have different roots and disjunct nodes and are therefore separated trees.

Parallel bitstreams might also share a node with another tree where the node becomes a root for another tree.

4.2.2. Mediated binary bitstreams

A path in a trito-structure might be decomposed into different binary parts this can happen as chaining or as overlapping parts.

$$(01120211002) \xrightarrow{\text{decomposition}} \begin{pmatrix} \mathbf{A:}\ 011 / 12 / 202 / 211 / 100 / 02 \\ \mathbf{B:}\ 011 / 112 / 202 / 211 / 1100 / 002 \end{pmatrix}$$

$$\mathbf{s}^{(3,1)} \longrightarrow \boxed{\begin{matrix} S1: (0, 1; \#) \\ S2: (1, 2; \#) \\ S3: (0, 2; \#) \end{matrix}}$$

$\mathbf{s}_{A}^{(3,1)}$: 01120211002 (3-bitstream A) $\xrightarrow{\text{decomposition}}$

S1	0	1	**1**	-	-	-	-	**1**	0	**0**	-
S2	-	-	**1**	**2**	-	**2**	1	**1**	-	**-**	-
S3	-	-	-	**2**	0	**2**	-	-	-	**0**	2

⟶

S1 : 011 #100 #
S2 : #12 #211 #
S3 : #202 #02

⟶

S1—— - -   - S1 -
- - S2 - - S2 - -
—— -- S3 - - - S3

⟶ $S_{1.2.3.2.1.3}$

Given a word or bitstream in a 3-contextural arithmetic there are different possible representations of it.
In contrast, a binary word in a classical binary arithmetic has a single representation only. Binary words are not ambiguos in respect to their decomposition, polycontextural words are enabling different decompositions, and therefore different interpretations.

Trees, and especially binary trees, have a single root and a tree of separated nodes. Kenogrammatc trito-structures have different roots inside the system, and the distinction between nodes and roots is exchangeable. The graph of trito-stuctures represents at a first glance a kind of a n-ary tree. It might be defined as a tree with a monadic root and its iterative and accretive successors.

The chapter *"Towards a General Model of Polycontextural Computation"*, Paragrafs 1-3 of *SKIZZE*-0.9.5 (in German) is dealing explicitly with the concepts, metaphors and semi-formal constructions of "*strange*" trito-arithmetic behaviors, i.e. poly-events of computation.
http://www.thinkartlab.com/pkl/media/SKIZZE-0.9.5-medium.pdf

Because trito-graphs are not representing identical objects there are different interpretations of the nodes and roots possible.
One interesting interpretation is given by a mapping of *binary* numbers onto the trito-structure. This corresponds an interpretation of n-ary trito trees as a composition of different binary trees.
As a consequence of such a mapping the trito-tree becomes a structure with a chiastic interchange between roots and nodes. Binary trees might start at different loci, i.e. distributed roots, in the trito-structure.
R. Kaehr, The Abacus of Universal Logics
http://www.thinkartlab.com/pkl/lola/Abacus.pdf

Localization of the 'number' TZ in the trito-system

$T^{(3)}$ – *number sequence* (01120211002)

$$\begin{pmatrix}
1: & (0)_1 \\
2: & ((0),(1)_2) \\
3: & (((0),(1)),((0),(1)_4,(2))) \\
4: & (((0),(1)),((0),(1),(2)),((0),(1),(2)),((0),(1),(2)_{11}),((0),(1),(2))) \\
5: & (((0),(1)),((0),(1),(2)),((0),(1),(2)),\ldots,((0)_{32},(1),(2)),\ldots,((0),(1),(2))) \\
6: & (((0),(1)),((0),(1),(2)),((0),(1),(2)),\ldots,((0),(1),((2))_{93})),\ldots,((0),(1),(2)) \\
7: & (((0),(1)),((0),(1),(2)),((0),(1),(2)),\ldots,((0),(1)_{284},(2))),\ldots,((0),(1),(2)) \\
8: & (((0),(1)),((0),(1),(2)),((0),(1),(2)),\ldots,((0),(1)_{851},(2))),\ldots,((0),(1),(2)) \\
9: & (((0),(1)),((0),(1),(2)),((0),(1),(2)),\ldots,((0)_{2552},(1),(2))),\ldots,((0),(1),(2)) \\
10: & (((0),(1)),((0),(1),(2)),((0),(1),(2)),\ldots,((0)_{7655},(1),(2))),\ldots,((0),(1),(2)) \\
11: & (((0),(1)),((0),(1),(2)),((0),(1),(2)),\ldots,((0),(1),(2)_{22968})),\ldots,((0),(1),(2))
\end{pmatrix}$$

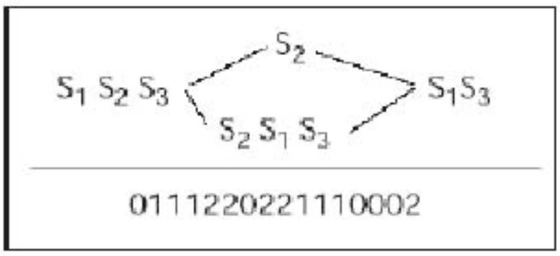
Decomposition of TZ

TZ = (01120211002)-decompositions

dec(01120211002)

$$\begin{bmatrix}
(011)_{1,2,4} \\
\quad (12)_{4,11} \\
\quad\quad (202)_{11,32,93} \\
\quad\quad\quad (211)_{93,284,851} \\
\quad\quad\quad\quad (100)_{851,2552,7655} \\
\quad\quad\quad\quad\quad (02)_{7655,22968}
\end{bmatrix}
\Bigg\|
\begin{bmatrix}
(011)_{1,2,4} \\
\quad (112)_{2,4,11} \\
\quad\quad (202)_{11,32,93} \\
\quad\quad\quad (211)_{93,284,851} \\
\quad\quad\quad\quad (1100)_{284,851,2552,7655} \\
\quad\quad\quad\quad\quad (002)_{2552,7655,22968}
\end{bmatrix}$$

Different decompositions of TZ
Example with different length of chains of subsystems of a decomposed trito-number TZ:
TZ = (01120000211002):
dec(TZ) =
[01/12/20/000/02/211/100/02] with $S_1S_2S_3S_1S_2S_1S_3$, l=8, r4=3
[01/12/20002/211/100/02] with $S_1S_2S_3S_1S_3$, l=6, r4=6



S1 = (0111) ==
S2 = (12) ==
S3 = (2022) ==
S2 = (2111) :: S2=(211), S1=(110), S3=(00)
S1 = (1000) =≠
S3 = (02) ==

The possibility to interpret a sequence in different ways enables an asymmetry between the construction and the destruction of the sequence. The way down has not to be the way up. Asymmetric inversions are possible. And obviously, a separation and reunion of the path of the sequence is accessible too.

Semiotics of 3-bitstreams:
signs= {1, 2},
nil-mark = {0, 1},
gap-mark= {#}.

A gap-mark is not a nil-sign like 0 in {0, 1} but marks at its contexture that there is no ocurrence of a sign, neither a mark nor a nil-martk, but that the missing signs happens in another contexture of the complexion.

4.2.3. Information-theoretic interpretation

Semiotics of 3 – bitstreams :

signs = $\{1, 2\}$,

nil – marks = $\{0, 1\}$,

gap – mark = $\{\#\}$.

Hence, the metric information is : $I = \mathrm{ld}\, N \left(\frac{\text{bit}}{\text{sign}}\right)$

Obviously, every binary number system gets it own logarithm and therefore its own information measure, thus

$$\text{bit}^{(3)}{}_{i\, \in\, (1d_{1.2.3})} = I^{(3)} = \left(\mathrm{ld}_1 N \amalg \mathrm{ld}_2 N \amalg \mathrm{ld}_3 N\right) = \mathrm{ld}_{1.2.3}\left(N_{1.2.3}\right)$$

$$K^{(3)} = K_{1.2.3} = \left(\frac{1}{\log_1 2}, \frac{2}{\log_2 2}, \frac{1}{\log_3 2}\right) = \frac{(1, 2, 1)}{\log_{1.2.3} 2}$$

Null

bitstream$^{(3)}$ =

S1	0	1	1	–	–	–	–	1	0	0	–
S2	–	–	1	2	–	2	1	1	–	|	–
S3	–	–	–	2	0	2	–	–	–	0	2
time	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11

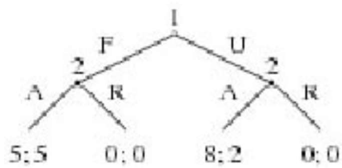
4.2.4. Bisimilarity of bitstreams

Polycontextural bitstreams are not properly measured by their static attributes of binary decision chains but much more by their behavior which allows to compare bitstreams in respect to the bisimilarity of their behavior.

4.3. Interpretations

4.3.1. Game theory

"The extensive form can be used to formalize games with some important order. Games here are often presented as trees (as pictured to the left). Here each vertex (or node) represents a point of choice for a player. The player is specified by a number listed by the vertex. The lines out of the vertex represent a possible action for that player. The payoffs are specified at the bottom of the tree.



"In the game pictured here, there are two players. Player 1 moves first and chooses either F or U. Player 2 sees Player 1's move and then chooses A or R. Suppose that Player 1 chooses U and then Player 2 chooses A, then Player 1 gets 8 and Player 2 gets 2." (WiKi, Game_theory)

It might be trivial but the example presumes that both players are playing the same game. That is, the continuation of the game by Player 2 at place 2 with A presumes a connection (also called glue or matching) between the action of Player 1 and Player 2 at the position

(U, 2) of the graph. This is certainly obvious but not yet formalized in the example nor in the axiomatics (below).
Obviously it can be omitted because both are playing the same game, and there is one game only to play.
Again, the situation is changing dramatically, if the game as such is complex and multi-layered, and in fact an interconnection or interplay of different games.

The category of journeys JOURN

A path trough a tree is part of the category of journeys JOURN. Journeys are classified into 4 different types of moving in a field of possibilities.
Paths don’t have gaps and ends. Journeys are offering strategies for jumps, redundancies and multiple end and beginnings for paths.
Path in a game theoretic tree are journeys of the first kind: there is a single path pre-given, find it.
Movements in trito-grammatic trees are of the third kind of moving in a field of possibility: paths have to be made by traveling.

The most reduced case for such a path finding concept is given with the binary interpretation of trito-trees: each root (beginning) playing the role of simultaneously being a vertex (node) or a root for a multitude of distributed binary (or n-ary) trees. Hence, incorporating ‘memror’ and ‘computation’.
http://www.thinkartlab.com/pkl/lola/Diamond Relations/Diamond Relations.pdf

Accepting this, totally new types of games are feasible.

It is a well known topic in *metaphorical* games and stories that the proponent disapears into another world - and that there is no path in the previous world which would lead to it. But this obvious metaphorical possibility is not yet formally realized by existing mono-contextural approaches to game theory, neither by recreational games nor by proof-theoretical games.

Mathematical formulation

A finite mono-contextural tree:

K = <V, v^0, T, p>

is a finite tree with a set of nodes V, a unique initial node $v^0 \in$ V, a set of terminal nodes $T \subset V$, $D = V \setminus T$ decision nodes and an immediate predecessor function p: V --> D on which the rules of the game are represented, ..."

A 3-tree is a dissemination of 3 single trees over the trito-structure:

3 – contextural tree

$$\mathcal{K}^{(3)} = \langle \mathcal{K}^{(1,1)}, \mathcal{K}^{(2,2)}, \mathcal{K}^{(3,3)} \rangle \text{ with}$$

$$\begin{array}{ccccc} \mathcal{K}^{(3,3)} \equiv & \mathcal{K}^{(1,1)} & \longrightarrow & \mathcal{K}^{(1,1)} \\ \downarrow & \updownarrow & & \updownarrow \\ \mathcal{K}^{(3,3)} \equiv & \mathcal{K}^{(2,2)} & \longleftarrow & \mathcal{K}^{(2,2)} \end{array} \text{ and}$$

$$\mathcal{K}^{(i,i)} = \langle\ \langle V, v^0, T, p \rangle^{i.i} \text{ for all } i = 1, 2, 3$$
$$\mathcal{K}^{(3)} = \langle V^{(3)}, v^{(3)0}, T^{(3)}, p^{(3)}, q^{(3)} \rangle .$$

For each contexture there is an *intra* – predecessor function p^i and *trans* – contextural functions q^i which are ruling the transitions between contextures.
The transition between contextures is chiastic and hence involved in memory / computation processes.

intra: $p^i : V^i \longrightarrow D^i, i = 1, 2, 3$
trans: $q^i : V^i \longrightarrow D^j, i \neq j$ with $\exists v^i \in V^i \equiv v^j{}_0 \in V^j$.

$\exists v^i \in V^i \equiv v^j{}_0 \in V^j$: chiasm$(v^i, v^j{}_0, i, j)$.

Hence, each node has successor and neighbor nodes.

A general polycontextural *forrest of trees – rhizom –* is a structure :
$< \mathcal{K}^{(m, n)}$, sop $>$ with mediated discontextural trees $\mathcal{K}^{(m, n)}$ and mappings
sop = {id, red, perm, repl, iter, bif} between contextures :
sop: $\mathcal{K}^{(m, n)} \longrightarrow \mathcal{K}^{(m, n)}$.

Rhizoms are forrests of coloured trees where vertices might become roots and roots vertices in other mediated constellations.

"The extensive *form, also called a* game tree*, is more detailed than the strategic form of a game. It is a complete description of how the game is played over time. This includes the order in which players take actions, the information that players have at the time they must take those actions, and the times at which any uncertainty in the situation is resolved.*
A game in extensive form may be analyzed directly, or can be converted into an equivalent strategic form."
http://www.cdam.lse.ac.uk/Reports/Files/cdam-2001-09.pdf

4.3.2. Emulations

"Memristors are not yet available for experimentation. Goal of this project is to EMULATE a memristor in a flexible way. The storage *and* controllable *resistor will be realized in* separate *devices, namely a simple microcontroller including an ADC and a tunable resistor."* Klaus Witrisal

With the proposed polycontextural model of memristive behavior a reasonable interaction between the two levels and their mediation is sketched. Memristive behavior as second-order behavior is not understood by a separated parallelism of the two functions, memory and control, of memristive systems. According to the interpretation of memristive systems as being of second-order, the levels have to be recognized (identified) as different functionalities and in the same effort as being mediated together too. Hence, memristive systems are demanding a new *architectonics* of computation.

4.3.3. Bifunctoriality of storage and control

A mathematical modeling, again, might start with the application of some features of monoidal category theory. The memristance of a ‘parallel’ distributed emulation of the functionalities of a memristor is defined by the interaction of ‘storage’ and ‘control’ in time over different ‘architectonical’ loci. Hence, memristance is modeled as an interaction of ‘storage’ and control'.

Emulation of memristance$^{(3,1)}$
The distribution scheme corresponds the *bifunctoriality* of parallel (par) and sequential (seq) processes in polycontextural categories:

(A par C) seq (B par D) = (A seq B) par (C seq D).
Applied to the example that reads as:

(storage par storage') seq (control par control') = (storage seq control) par (storage' seq control').

Interchangeability of memristance

$$\mathcal{U}_2 = \{\text{storage}_2,\ \text{control}_2\}$$

$$\mathcal{U}_1 = \{\text{storage}_1,\ \text{control}_1\}$$

$$\left(\mathcal{U}_1 \bigcap_{1.2} \mathcal{U}_2\right) \bigcap_{1.2.3} \mathcal{U}_3 = \emptyset$$

$$\mathcal{U}^{(3)} = \left(\mathcal{U}_1 \amalg_{1.2} \mathcal{U}_2\right) \amalg_{1.2.3} \mathcal{U}_3 :$$

$$\begin{bmatrix} \text{control}_1 & \text{control}_2 \\ \text{storage}_1 & \text{storage}_2 \end{bmatrix} :$$

$$\begin{bmatrix} \begin{pmatrix} \left((\text{storage}_1)_{1.0.0}\right) \\ \amalg_{1.2} \\ (\text{storage}_2)_{0.2.0} \end{pmatrix} \\ \amalg_{1.2.3} \\ (\text{memristance}_1)_{0.0.3} \end{bmatrix} \circ_{1.2.3} \begin{bmatrix} \begin{pmatrix} (\text{control}_1)_{1.0.0} \\ \amalg_{1.2} \\ (\text{control}_2)_{0.2.0} \end{pmatrix} \\ \amalg_{1.2.3} \\ (\text{memristance}_2)_{0.0.3} \end{bmatrix} =$$

$$\begin{bmatrix} \begin{pmatrix} (\text{storage}_1) \circ_{1.0.0} (\text{control}_1) \\ \amalg_{1.2} \\ (\text{storage}_2) \circ_{0.2.0} (\text{control}_2) \end{pmatrix} \\ \amalg_{1.2.3} \\ (\text{memristance}_1) \circ_{0.0.3} (\text{memristance}_2) \end{bmatrix}$$

4.3.4. Logic of information

The Boolean algebra of information has no laws corresponding the law of bifunctorial interchangeability. Bifunctoriality or the law of interchangeability (Vertauschungsgesetz, Hasse) is not as such a provable theorem in Boolean algebra like the law of distributivity.

Nevertheless there is an interesting *logical* antecessor in Leibniz' *Praeclarum theorema* to the categorical bifunctoriality of composition and yuxtaposition of parallel and serial processes in a unique universe of discourse.

"An illustrious example of a propositional theorem is the *praeclarum theorema*, the admirable, shining, or *splendid theorem* of Leibniz.

"If a is b and d is c, then ad will be bc.
This is a fine theorem, which is proved in this way:
a is b, therefore ad is bd (by what precedes),
d is c, therefore bd is bc (again by what precedes),
ad is bd, and bd is bc, therefore ad is bc. Q.E.D."
(Leibniz, Logical Papers, p. 41).
http://mywikibiz.com/Futures_Of_Logical_Graphs#Praeclarum_theorema

Leibniz' Praeclarum Theorema in propositional formulation:

$$\left(\left(a \Longrightarrow b\right) \wedge \left(d \Longrightarrow c\right)\right) \Longrightarrow \left(\left(a \wedge d\right) \Longrightarrow \left(b \wedge c\right)\right).$$

With $\{a, b, c, d\} \in U$

we might translate the logical formula into the categorical formula for bifunctoriality:

$$\left(\left(a \circ b\right) \otimes \left(d \circ c\right)\right) \Longrightarrow \left(\left(a \otimes d\right) \circ \left(b \otimes c\right)\right).$$

Bifunctoriality of the Praeclarum Theorema

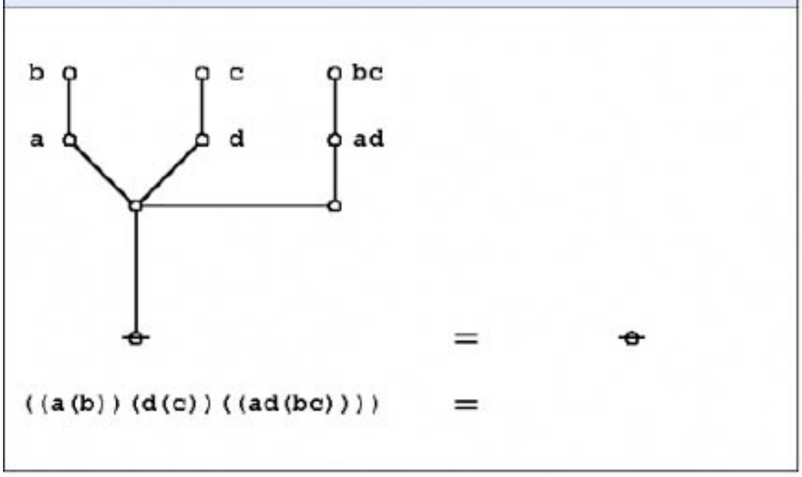
$\{a, b, c, d\} \in U$

$$\begin{bmatrix} b_1 & c_2 \\ a_1 & d_2 \end{bmatrix} :$$

$$\begin{pmatrix} a_1 \\ \otimes \\ d_2 \end{pmatrix} \circ \begin{pmatrix} b_1 \\ \otimes \\ c_2 \end{pmatrix} = \begin{pmatrix} \left(a_1 \circ b_1\right) \\ \otimes \\ \left(d_2 \circ c_2\right) \end{pmatrix}$$

There is a nice proof of the formula in the framework of Peirce's logical graphs and some application of the Laws of Form from G. Spencer Brown at Jon Awbrey's Knol "*Praeclarum Theorema*" and proofwiki.

Praeclarum Theorema (Leibniz)



http://www.proofwiki.org/wiki/Praeclarum_Theorema

Semiotic interpretation by John F. Sowa is given here:
http://www.jfsowa.com/peirce/remark.pdf

Distributed splenditity

Leibniz' splendid theorem is set in a uniform single universe of objects, hence