Graphematic System of Cellular Automata

Short characterization of cellular automata by the 9 graphematic levels of inscription

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Abstract

As a further specification of the *"overview of morphic cellular automata"*, described before, a graphematic classification of the inscriptional systems shall be introduced and applied to different types of cellular automata. The combinatory aspects of the classification of mappings are based on Dieter Schadach's work, this work is cited but not fully reconstructed, proofs are omitted and referred to the original work.

1. A system of types of inscriptions

1.1. Schadach's complete classification of mappings

1.1.1. Systems of classifications

When Gotthard Gunther was forced in the early '60s to abandon his multiple-valued approach in favor of a kenogrammatic radicalization of his ideas towards a trans-classical logic, the question immediately raised about what kind of mathematical system could be behind his kenogramatics he introduced intuitively.

Of the different approaches at that time at the BCL (Biological Computer Laboratory, Urbana, Ill., USA) to develop a mathematical foundation to support Gunther's trans-classical studies towards kenogrammatics, the mathematical work of his assistant Dieter Schadach was of great importance. Schadach, supported by Heinz von Foerster and Alex Adrew, developed a general method of classification of mappings between sets which offered methods to determine combinatorial questions specific to the trans-classical project and was able to open up different new strategies and methods of trans-classical keno- and morhogrammatics and speculations about the deep structure of time and logic for evolving/emanating historical systems.

One point was to question: *How many different levels exist in the classification of the whole system of kenogrammatics?*

The other important question was: *What is replacing the atomic elements of sets and sequences in the kenogrammatic settings, especially on the level of trito-grams?*

A further important question appeared. Given the classification: *How many sign sequences are represented on a level of such a classification?* Especially, how many sign sequences are represented by an arbitrary tritogram?

Many other serious questions raised especially with the combinatorics of morphogrammatic compound systems for the foundation of multi-valued trans-classical logics. This question was profoundly answered with the dissertation of H. S. H. Na, *"On structural analysis of many valued logics"*, 1964.

All the approaches to further analysis by Heinz von Foerster, Na and Schadach had been based naturally on set-theoretical methods of mappings between sets, reasonable for combinatory studies, therefore, all the approaches had been based semiotically on the identity of its elements. The basics of this approach are defined by identical signs delivered as sets, multi-sets, heaps or different kinds of sequences of identical signs.

After all this combinatorial work had been done and has found a readable reconstruction and implementation in the book "*Morphogrammatik*" (Kaehr, Mahler) it seems to be appropriate to invent a more *dynamic* approach to classification. Instead of starting with semiotic identity it could be interesting to find a beginning in any other conceptual level of inscription, and therefore establishing a dynamical system of classification of scriptural praxis.

A first step towards a 'beginning' of a semiotic economy not based on identity is proposed by the S*tirling Turn,* which is starting with tritograms, i.e. with keno- and morphograms of the trito-structure of kenogrammatics.

Monomorphies

The question about a replacment of semiotic atoms, elements or objects of the semiotic identitive level of classification was given by Schadach's concept of *monomorphies* of morphograms for the trito-level of kenogrammatic systems.

Gunther himself introduced 3 kenogramamtic levels of classification, *trito-*, *deutero-* and *proto-*structure, what produced an interesting debate because

Schadach's formal classification of mappings between sets contained 8 different levels, 4 specific independent levels plus *identity* and *cardinality* building together with the 3 mixed levels a similar structure like the 10-fold Sephirot of Kabbalah. On the other hand, Gunther's classification got just 3 kenogrammatic levels plus identity and cardinality.

The solution to this discrepancy is quiet simple. Both are arguing on different classificational principles. Gunther on strict kenogrammatic properties involving the base set, A/Kern μ_i, of mappings only, while Schadach's classification is based on on the whole system of set-theoretical mappings.

Schadach's classification contains additionally to Gunther's 3 independent levels, proto-, deutero- and trito-structure, 2 further independent levels which didn't get a logical or philosophical interpretation.

Indicational semiotics
An interesting interpretation of a non-kenogrammatic level in Schadach's system of classification became accessible with Mathias Varga von Kibéd and Rudi Matzka's semiotic understanding of George Spencer-Brown's *Calculus of Indication* as a *'topology-invariant"* semiotic system. Therefore, this level is based on *identity* but is abstracting from the *order* of its identical signs. This corresponds to Schadach's independent level $B^A/_{III}$ with the property δ and the cardinality $\binom{n+m-1}{n}$. The name of this number series is not yet mentioned.

"Diese drei Unterscheidungsmerkmale, also
a) die Verwendung leerer Symbole
b) die topologisch invariante Notation (und damit die Reihenfolgeunabhängigkeit der Argumente des Operators) und
c) die Nicht-stelligkeit des Operators,
erlauben prinzipiell nicht, von einer vollständigen Isomorphie der Kalküle von Spencer Brown mit gängigen formalen Systemen zu sprechen." (Kibed, Matzka)

The fact, that the indicational level is proven by Schadach as an *independent* classification in the system of classification of mappings gives the adventure of George Spencer Brown and his *Laws of Form* a scientific foundation and legitimation not achieved by other attempts to justify the calculus of indication.

It also makes clear that any attempt to domesticate indication by propositional logics and linguistic-based semiotics obsolete. But it also shows at the same time clearly the radical limitation of the *Laws of Form* to a “binary” concept of indication which is denying the available richness of indicational symbolizations.

What was not seen by Brown and the Brownians is the even legitimate “complementary” system of symbolization or indication by the system based on the Mersenne numbers

Mersenne semiotics

The second *independent* level not reflected in Gunther’s kenogrammatics is the level $B^{A}{}_{/VII}$ with the property ε and the cardinality $M_n = 2^n - 1$. This level corresponds combinatorically to the *Mersenne* numbers. It delivers the inscriptual base for a Mersenne semiotics, logic and arithmetics complementary to the indicational semiotics, logic and arithmetics still to be elaborated. Mersenne numbers and perfect Mersenne numbers played an important role in Gunther’s studies (1968) towards a reflectional theory of history.

Comparision Mersenne-Brown

A kind of a duality holds between the indicational semiotics in the sense of the George Spencer Brown’s calculus of indication and the semiotics of a possible Mersenne calculus of distinction.

The calculus of indication is topoloy-invariant, i.e. invariant under permutation, and identity-variant for atomic elements.

The Mersenne calculus is topology-variant, i.e. variant under permutation, and identity-invariant for non-atomic elements.

Indicational rules: (aa)$\neq_{ind}$ (bb), (ab) $=_{ind}$(ba); cardinality $Ind_{(n,\, m)} = \binom{n+m-1}{n}$,

Mersenne rules: (aa) $=_{mers}$ (bb), (ab) $\neq_{mers}$ (ba); cardinality $M_n = 2^n$-1.

Indication:	**Mersenne:**
J1: { }{ } = { }	M1: { }{ } = ∅
J2: {{ }} = ∅	M2: {{ }} = { }.
m=n=2	n=2
card(ind(2, 2)) = 3	card(Mers(2)) = 3
(aa), (bb); (ab).	(aa); (ab), (ba).

$m = 2,\ n = 4$

card(Ind(2, 4)) = 5

{aaaa, bbbb; aaab, abbb, aabb}.

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card (Mers (2, 4)) = 15 :
{aaaa; aaab, aaba, abaa, aabb, abab, abba, abbb;
       bbba, bbab, babb, bbaa, baba, baab, baaa}.
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Completeness

Schadach proves the *completeness* of his classification system. There exists no other classifications of independent and mixed mappings over B^A.

Therefore, Gunther's kenogrammatic system is, in respect to kenogrammatics, complete too. There are no further independent or mixed genuine kenogrammatic classifications over B^A.

On the base of this complete system of classification arbitrary new special cases and mixtures are trivially possible.

Obviously, the common condition of the Schadach's classification is the preservation of the "length" of the mappings and the exclusion of any metamorphic properties of bisimilarity between mappings of different "length".

Mixed semiotics

All further distinctions like trito-, deutero- and proto-structure of *commutative* and *partitive* semiotics contain a mix of concepts of identical signs and of kenogrammatics.

Schadach's dependency graph starts with identity (i) and ends with the cardinal equivalence (c).

Implicational orders

Implicational order

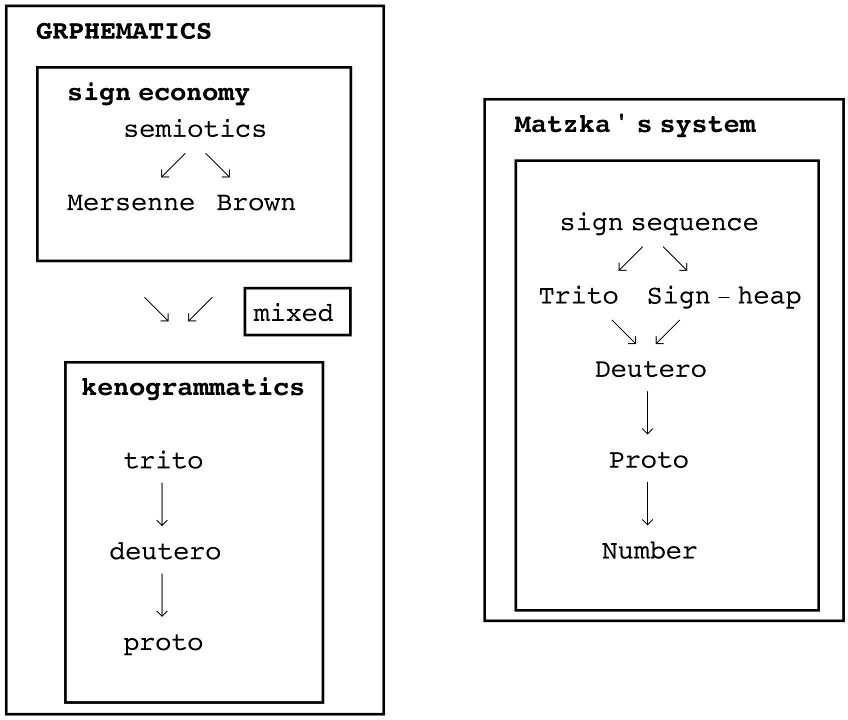
$$(i) \longrightarrow (a,\ d) \begin{matrix} \nearrow (d, e) \longrightarrow (a) \longrightarrow (b) \searrow \\ \searrow (d) \longrightarrow (b, e) \longrightarrow (e) \nearrow \end{matrix} (g) \longrightarrow (c)$$

Implicational order

$$(i) \begin{matrix} \nearrow (a) \longrightarrow (b) \searrow \\ \searrow (d) \longrightarrow (e) \nearrow \end{matrix} (g) \longrightarrow (c)$$

a = trito, b = deutero, g = proto – kenogrammatics

d = indicational (sign – heap), e = Mersenne
i = identity semiotics, c = cardinality,
$(a, d), (d, e), (b, e)$ = mixed systems



1.1.2. Schadach's system of classification

Let A and B nonempty finite sets $A = \{a_1, a_2, \ldots, a_n\}$ and $B = \{b_1, b_2, \ldots, b_m\}$
Let B^A denote the set of allmappings from A to B,

$B^A = \{\mu \mid \mu: A \longrightarrow B\}$.

This is elaborated at: *Morphogrammatik*.

How to construct monomorphies mathematically?
The question: *What replaces atomic signs in a kenogrammatic pattern (morphogram)?* Is answered by Schadach with the introduction of *monomorphies* of morphograms.

From a mathematical point of view, monomorphies are *partitions* of mappings. This is well elaborated by [Schadach 1967]. The procedure to build monomorphies out from morphograms, as it is mathematically defined by Schadach's approach, shall be called *monomorphic decomposition*.

" Let A and B be non – empty finite sets,
$A = \{a_1, a_2, \ldots, a_n\}$, $B = \{b_1, b_2, \ldots, b_m\}$.
Let denote B^A the set of all mappings from A to B,
$B^A = \{\mu \mid \mu : A \longrightarrow B\}$, $\text{card}\, B^A = (\text{card}\, B)^{\text{card}\, A} = m^n$.

Corollary 1.

If $I_x = \emptyset$, then $[\mu]_\emptyset = B^A - \bigcup_{i \in I} R_i$ and

if $I_x = I$, then $[\mu]_I = \bigcap_{i \in I} R_i$.

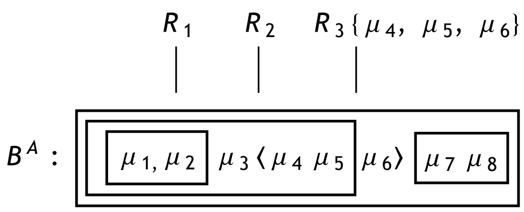
Corollary 2

By Theorem 1, we get *a* mapping from the set of all families of subsets of B^A onto the set of all partitions of B^A.

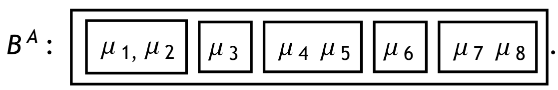
Example.

Let be $B^A = \{\mu_1, \mu_2 \ldots, \mu_8\}$ and the family of subsets $\{R_i \mid i \in I = \{1, 2, 3\}\}$ where

$R_1 = \{\mu_1, \mu_2\}$,
$R_2 = \{\mu_1, \mu_2, \mu_3, \mu_4, \mu_5\}$,
$R_3 = \{\mu_4, \mu_5, \mu_6\}$.



I_x	$[\mu]_{I_x}$
$\emptyset$	$B^A - (R_1 \cup R_2 \cup R_3) = \{\mu_7, \mu_8\}$
$\{1\}$	$R_1 - (R_2 \cup R_3) = \emptyset$
$\{2\}$	$R_2 - (R_1 \cup R_3) = \{\mu_3\}$
$\{3\}$	$R_3 - (R_1 \cup R_2) = \{\mu_6\}$
$\{1, 2\}$	$(R_1 \cap R_2) - R_3) = \{\mu_1, \mu_2\}$
$\{1, 3\}$	$(R_1 \cap R_3) - R_2 = \{\mu_7, \mu_8\}$
$\{2, 3\}$	$(R_2 \cup R_3) - R_1 = \{\mu_4, \mu_5\}$
I	$R_1 \cap R_2 \cap R_3 = \emptyset$



(Dieter J. Schadach, BCL Report No. 4.1, August 1, 1967)

1.1.3. Table of graphematic systems

Authors	Nummer	Sign Types	Property	Cardinality
Gunther, Schadach, von Foerster	IV (α)	Trito – Keno	retrograde recursive, bifunctorial	$\sum_{k=1}^{M} S(n, k)$
Gunther	VI, (β)	Deutero – Keno	associative	$\sum_{k=1}^{M} P(n, k)$
Gunther	VIII, (γ)	Proto – Keno	commutative	$\min(\mathrm{cardA}, \mathrm{cardB})$
Spencer Brown, Varga von Kibed, Matzka	III, (δ)	Indicational semiotics	commutativ – identitive	$\binom{n+m-1}{n}$
Mersenne	VII (ε)	Mersenne semiotics	partitive identitive	$M_n = 2^n - 1$
Leibniz, Hermes, Schroter	0 (i)	Semiotics	identitive associative, recursive	m^n
Graphematics	II (α, ϵ)	mixed	trito – partitive semiotics	$\sum_{k=1}^{M} S(n, k)\binom{m}{k}$
Graphematics	I (α, δ)	mixed	trito – commutative semiotics	$\sum_{k=1}^{M} S(n, k)\binom{n+m-1}{n}$
Graphematics	V (β, ε)	mixed	deutero – partitive semiotics	$\sum_{k=1}^{M} P(n, k)\binom{m}{n}$

Proömik und Disseminatorik in: DISSEMINATORIK. Theorie polykontexturaler Systeme,
http://www.thinkartlab.com/pkl/media/DISSEM-final.pdf

1.1.4. System of independent characteristics

Interestingly there are two different aspects of classification, one class are independent and the other class are dependent, i.e. mixed classififications.

The idependent classifications are divided into two different types: *kenogrammatic* and *semiotic*.
One classification of the independent type is delivering the kenogrammatic classifications of *trito-deutero-* and *proto*-structure (α, β, γ).

The other classifcation of the independent type is delivering an independent semiotic specification of the '*indicational*' (III, δ) and the '*partitive*' (VII, ε) classifications. Both chains start with *identity* (0, ι) and end in the *trito*-structure (IV, α).

The kenogrammatic classifications are clearly depending on the classification of the target sets of their mappings, i.e. A/Kern μ of the mapping μ A $\longrightarrow$ B, with $\mu \in B^A$.

The other kinds of classification are achieved by a *mix* of the independent criteria.
I = (α, δ), II = (δ, ϵ) and V = (δ, ε).
Until now there are no scriptural systems known with the characteristics of the mixed graphematic systems.

Obviously, the classification *semiotics* (0, ι) got a highly complete elaboration.

From the kenogrammatic systems, the trito-structure (IV, α) got some elaboration. Specific studies to deutero- and proto-systems are not yet available, probably because it is believed that their structures are quiet simple compared to the trito-structure.

From the semiotic abstractions (III, δ) and (VII, ε), only (III, δ) got a extensive study in the disguise of the Calculus of Indication of Spencer Brown's Laws of Form. But a specific graphematic study of the *Laws of Form* is still lacking.
It seems that the *Calculus of Indication* appears as a surface structure pointing to the graphematic structure of 'commutative' and 'topologically' invariant properties of graphematic systems of inscription with the crucial restriction to a 2-element alphabet.

The semiotic abstraction (VII, ε) which is connected to *Mersenne* numbers seems to reflect a kind of a dual symbolization to the *indicational* system (III, δ). This observation is not yet elaborated enough to make any reasonable comments.
http://www.rudolf-matzka.de/dharma/kenogrammatik.pdf

A general approach to Calculi of Indication for arbitrary elements had been proposed by the introduction of the quadralectic Diamond Calculi.
http://www.thinkartlab.com/pkl/media/Diamond%20Calculus/Diamond%20Calculus.html

Other formats

For people who are into theorems it is proven that Indicational semiotics and

Mersenne semiotics are *independent* semiotics in the graphematical system of inscription. Further more it is proven that Indicational semiotics and Mersenne semiotics share a kind of duality or complementarity in the graphematic system of inscription (symbolization).

Theorem-I
Indicational Calculi are semiotically independent systems.

Theorem-II
Mersenne Calculi are semiotically independent systems.

Lemma-1
George Spencer Brown's Calculus of Indication is a special case of Indicational Calculi.

Lemma-2
The Calculus of Indication is the smallest possible indicational calculus.

Proofs are left to the reader.

Summary
There are
1. 3 kenogrammatic systems: *trito*, *proto* and *deutero*.
2. 3 identitive systems: *semiotics*, partition (*Mersenne*), *indication* (Spencer Brown)
3. 3 mixed identical-kenomic systems: *trito-partitive*, *trito-commutative* (trito-Brown), *deutero-partitive*.

1.1.5. Interdependencies and derivations

Indicational systems in the form of George Spencer Brown's Calculus of Indication had been studied in extenso by Louis Kauffman and got a especial reception in German sociological system theory (Luhmann) but mainly under the restriction of speculative applications, avoiding any further formal elaborations.

The graphematic question is: *Are there other graphematic systems like trito-kenogrammatics or identitive semiotics deducible from the indicational approach?*

Indication is indicated as a independent level of inscription. But are the other independent level achievable with the means of indication, say in the sense, as indiction is achievable from the trito-structure?

Combinatorically, the question is: *How can we abstract, say from the trito-formula* $\sum S(n, k)$*to the indicational formula* $\binom{n+m-1}{n}$?

Descriptively, how to define [aaa] $=_{trito}$[bbb] with indication which states (aaa) $\neq_{ind}$(bbb), but (abb) $=_{ind}$ (bba)?

It seems to be easy to define indication from the position of the trito-structure.
[aaa] $=_{trito}$[bbb] $\Rightarrow_{cristallization}$ {(aaa), (bbb)}$_{SEM}$ and (aaa)$\neq_{ind}$ (bbb).
With [ab} $=_{trito}$[ba], the indicational <ab> = <ba> holds trivially.

In respect of the *cardinality* of the mappings there is a direct implicational order between *indication ⇒ deutero ⇒ trito*.

1.2. The Stirling Turn for morphogrammatics

1.2.1. Abstractions, concretizations and mixtures

According to the newly introduced *Stirling Turn*, the dependency graph might be changed with (trito) at the top and therefore involving a different approach to the application of abstractions over trito-structures.
This kind of classification is related also to the complexity of the *properties* of the classification levels and not just to the *cardinality* of repeatable elements like it is the case for the morphism approach of a classification starting with the level of identity.

In this sense, the level of tritograms seems to be the most complex with interesting properties like *retrograde* recursivity and element- or object-independence. Morphograms of the trito-level are composed by *monomorphies* and not by atomic elements of a sign repertoire. The structure of morphograms is not limited to linear sequences of monomorphies. Monomorphies are enabling naturally to study *metamorphic* transformations and *bisimilarity* of behaviors.

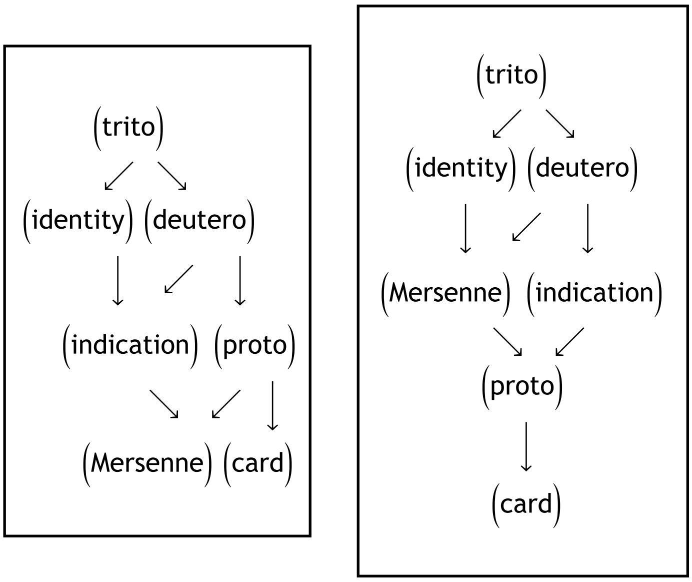
The *Stirling Turn* takes into account the fundamental property of the *memristivity* of matter and its inscription in the medium of symbolization realized by the trito-structure of kenogrammatics.

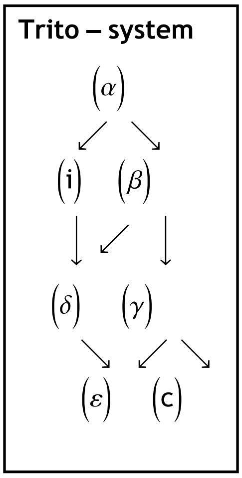
Identity comes then as a ‘crystallization' (objectivation) of tritograms, while the *deutero*- and *protograms* are introduced as successive direct abstractions from the genuine trito-level. This objectivation of tritograms to identities is eliminating fundamental properties of tritogrammatic level, like retrogradness, metamophy and bisimilarity. Such properties might be reconstructed secondarily on the base of identitive semiotics but are no more of any fundamental importance.

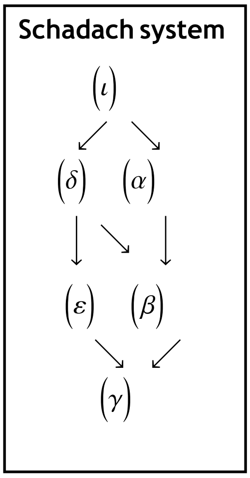
An interesting mix of *properties*, identity and deutero, is introduced with the *indicational* level of abstraction which doesn't therefore belong to the strictly kenogrammatic system. The indicational level is independent, the same holds for the Mersenne level.
But the properties might be understood as a mix of the *deutero*- and the *identity*-level where the deutero-levels entails properties from the trito-level too. Deutero-properties, (aab) $=_{Deutero}$ (aba)$=_{Deuero}$ (baa) and (aaa) = $_{Deutero}$(bbb), hold for the indicational commutativity, i.e. (ab) $=_{ind}$ (ba) and (aa) $\neq_{ind}$(bb), and for the Mersenne-level with (aa) $=_{MERS}$ (bb) and (ab) $\neq_{MERS}$ (ba).

Cardinality is an obvious further abstraction of the proto-structure of morphograms.

There is no new dogmatism involved with the *Stirling Turn,* which takes tritograms as an initial level for the development of all graphematic levels of inscription. With a change of focus to other prevalences other kinds of classifications might be reasonable. In contrast, the semiotic approach to classifications, which is included in the graphematic approach as a special kind, is not able to offer a dynamic approach to classificatory systems with changing standpoints or contexts of prevalences.







Type	**Property**
(α) = trito	keno-retrograde recursivity, monomorphies,
(β) = deutero	keno-partitive
(γ) = proto	keno-commutativity, distributive, associative
(ι) = identity	recursivity, associative, monoid
(δ) = indicational	indentive "topological" commutativity
(ε) = Mersenne	identitive partitive
(c) = cardinality	

1.2.2. Graphematic CAs and equivalence classes of CCAs

According to the 10 graphematic types of inscriptions, ten fundamentally different types of cellular automata are accessible for definitions. On each inscriptional level of graphematics a specific kind of cellular automata shall be defined.

In strict contrast, a *conservative* interpretation of the classification based on equivalence relations is achieved with a classification of the *data set* of identitive cellular automata B^A into equivalence classes of CAs, therefore, with the start of classical concept of CAs as mappings we
get:

$$\text{classCA}^{(m,n)} = \text{CA}^{(m,n)}{}_{/\text{class}}$$

$$\text{class} = \{\text{I, II, ..., X}\}$$

CA = (grid, data, rules)

$CA_{/class}$ = (grid, $data_{/class}$, rules)

With trito-systemes as a beginning of the graphematic system of classification is not anymore conserned primarily with the *cardinality* of the classes but with the system of there *operational* properties. Classification or systematization becomes then a mix of *abstraction* and *concretization* (crystallisation) of kenomic patterns.

Example for trito – version

$m = 2,\ n = 3$

$$\left[abb\right]_{trito}$$

$$\swarrow \quad \searrow$$

$$\left(abb\right); \quad \left\{abb\right\}, \left\{bba\right\}, \left\{bab\right\};$$
$$\left(baa\right)_{ident} \quad \left\{baa\right\}, \left\{aab\right\}, \left\{aba\right\}_{deutero}$$

$$\downarrow \quad \swarrow \quad \downarrow$$

$$<abb>,\ <bba>,\ <bab>_{ind}; \quad \left\{abb\right\}, \left\{bba\right\}, \left\{bab\right\};$$
$$<baa>,\ <aab>,\ <aba>_{ind} \quad \left\{baa\right\}, \left\{aab\right\}, \left\{aba\right\}_{proto}$$

$$\searrow \quad \swarrow$$

$$\left(abb\right), \left\{bba\right\}, \left\{bab\right\};$$
$$\left(baa\right), \left\{aab\right\}, \left\{aba\right\}_{Mersenne}$$

1.3. Category-theoretic classifications

Classifications based on different operators are delivering second-order concepts like the *bisimilarity* of the behavior of operational systems.

How does the distinctions of equality, equivalence, similarity and bisimilarity, introduced for different cellular automata and semi-Thue systems, fit into the classification of notation systems?

The partition, introduced by Dieter Schadach, is based on set-theoretic considerations for a complete system of partitions. Certainly, the concept of bisimilarity is not directly covered by such an attempt to a classification of set-based mappings.

What is the difference to semiotic abstractions? The kenomic abstraction of bisimilarity happens over the operators and not over the sign sets like for semiotic systems. Equivalence classes in semiotic systems are build over sets of

signs and not over the operations on signs. Hence, the kenomic abstraction of bisimilarity is a kind of a second-order abstraction.
http://www.thinkartlab.com/pkl/media/Web_Mobility/Web_Mobility.pdf

2. Graphematics of inscriptional systems

2.1. Morphism-based classification

2.1.1. Equality: Semiotics (B^A)

$$\text{card } B^{A} = m^{n}$$

where $n = \text{card } A$, $m = \text{card } B$,
$M = \min\{\text{card } A,\ \text{card } B\} = \min\{m,\ n\}$.

$$\sum_{k=1}^{M} \frac{m!}{(m-k)!} S(n,\ k) = \mathrm{m}^{n}.$$

Sem(2, 3) = 2^3 = 8
Alphabet = {a, b}
2^3: {aaa, bbb, aab, aba, abb, baa, bab, bba}.

B^A : 2^4 = 16: {aaaa, aaab, aaba, aabb, abaa, abba, abab, abbb, bbbb, bbba, bbab, bbaa, babb, baab, baba, baaa}.

Sem(3, 2) = 3^2 = 9
Alpabet= {a, b, c}
{aa, bb, cc, ab, ac, ba, bc, ca, cb}

2.1.2. Keno-Equivalence: Trito (IV)

$$\text{card}(\text{Trito}(m,\ n)) = \sum_{k=1}^{M} S(n,\ k)$$

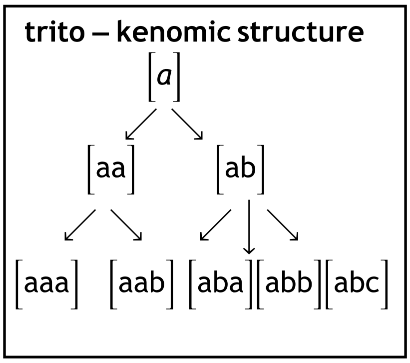
where $n = \text{card } A$, $m = \text{card } B$, $M = \min\{\text{card } A,\ \text{card } B\}$

$$\sum_{k=1}^{2} S(n,\ k) = 1 + 7 = 8$$

$$\textbf{IV.} = \{\text{aaaa, aaab, aaba, abaa, baaa, aabb, abab, abba}\}.$$

$$\sum_{k=1}^{4} S(n,\ k) = 1 + 7 + 6 + 1 = 15$$

$$\text{Trito}(4,\ 4) = \{\text{aaaa; aaab, aaba, abaa, aabb, abab, abba, abbb; aabc, abac, abbc, abca, abcb, abcc; abcd}\}.$$



Partitions with localization and repetitions.

2.1.3. Keno-Equivalence: Deutero (VI)

$$\text{card}(\text{Deutero}(m,\ n)) = \sum_{k=1}^{M} P(n,\ k)$$

where $n = \text{card}\,A,\ m = \text{card}\,B,\ M = \min\{\text{card}\,A,\ \text{card}\,B\}$

Partitions (without localizations, repetitions)

$$\sum_{k=1}^{4} P(4,\ k) : 1 + 2 + 1 + 1 = 5$$

$$\sum_{k=1}^{4} P(4,\ k) : \{\text{aaaa, aaaab, aabb, aabc, abcd}\}.$$

$$\sum_{k=1}^{2} P(4,\ k) : 1 + 2 = 3.$$

$$\textbf{VI.} = \{\text{aaaa, aaaab, aabb}\}.$$

Sloane’s A000041 a(n) = number of partitions of n (the partition numbers).
1, 1, 2, 3, 5, 7, 11, 15, 22, 30, 42, 56, 77, 101, 135, 176, 231, 297, 385, 490, 627, 792,

1002, 1255, 1575, 1958, 2436, 3010, 3718, 4565, 5604, 6842, 8349, 10143, 12310, 14883, 17977, 21637, 26015, 31185, 37338, 44583, 53174, 63261, 75175, 89134

"The subject of partitioning integers is very rich and quite deep."

http://www.artofproblemsolving.com/Resources/Papers/LaurendiPartitions.pdf

Programming aspects of partitions.

http://programmingpraxis.com/2011/04/15/partition-numbers/

2.1.4. Keno-Equivalence: Proto (VIII, γ)

$$\text{card}\,(\text{Proto}\,(\text{m},\ \text{n})) = \min\,(\text{cardA},\ \text{cardB})$$

VIII. = {aaaa, aaab}.

2.1.5. Indicational (III, δ)

$$\text{card}(\text{Ind}(m,\ n)) = \binom{n + m - 1}{n}$$

where $n = \text{card}\,A,\ m = \text{card}\,B,\ M = \min\{\text{card}\,A,\ \text{card}\,B\}$

Pascal (Yang Hui) combinations

$$\binom{m}{n} = \frac{m!}{n!\,(m - n)!}$$

Examples

$m = n = 2:\ \binom{2 + 2 - 1}{2} = \binom{3}{2} = 3$

Alpabet = $\{a,\ b\}$

Ind (2, 2) = {aa, ab, bb} .

$m = 2,\ n = 3:\ \binom{3 + 2 - 1}{3} = \binom{4}{3} = 4$

Alphabet = $\{a,\ b\}$

card(Ind(3, 2)) = {aaa, aab, abb, bbb}.

$m = n = 3$

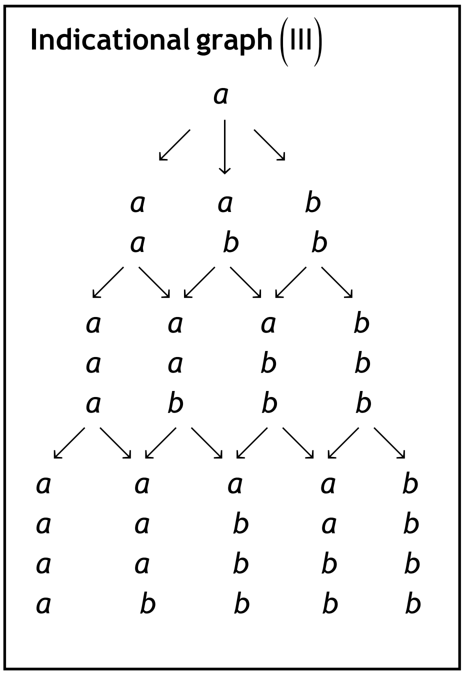
card(Ind(3, 3)) = 10

Ind(3, 3) = {aaa, bbb, ccc, aab, aac, abb, acc, bcc, bbc, abc}.

$m = 2,\ n = 4$

card(Ind(2, 4)) = 5

III. = {aaaa, bbbb; aaab, abbb; aabb }



2.1.6. Partitive identitive semiotics : VII (ε)

A Mersenne number, not with primes, is a number of the form :

$$M_n = 2^n - 1$$

1, 3, 7, 15, 31, 63, 127, 255, (Sloane's A000225),

http://mathworld.wolfram.com/MersenneNumber.html

Mersenne numbers.

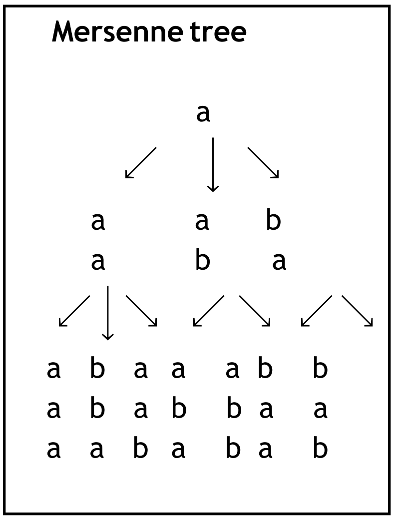
$2^2 - 1 = 3$: {aa, ab, ba}, (eq: aaaa, bbbb, aaab).

$2^3 - 1 = 7$: {aaa, aab, aba, abb, bba, bab, baa}

$2^4 - 1 = 15$: {aaaa; aaab, aaba, abaa, aabb, abab, abba, abbb;
bbba, bbab, babb, bbaa, baba, baab, baaa.

VII. = {aaaa, bbbb, aaab}.

$(aa) =_{MERS} (bb),\ (ab) \neq_{MERS} (ba)$



2.1.7. Trito-partitive semiotics (II, α, ε)

$$\operatorname{card} B^{A} /_{\text{II}} = \sum_{k=1}^{M} S(m,\ k)\binom{m}{k}$$

where $n = \operatorname{card} A,\ m = \operatorname{card} B,\ M = \min\{\operatorname{card} A,\ \operatorname{card} B\}$

Example : m = n = 4

$$\textbf{IV.} = \sum_{k=1}^{4} S(m,\ k)\binom{m}{k}:$$

$$\textbf{IV.}\ \text{Trito}(4,\ 2) = \sum_{k=1}^{2} S(4,\ k) = 8$$

$$\text{xy.} \quad \binom{m}{k} : \binom{2}{2} = 1$$

$$\textbf{II.} = \sum_{k=1}^{2} S(m,\ k)\binom{m}{k} : 8 + 1 = 9$$

$$\textbf{II.} = \{\text{aaaa, bbbb; aaab, aaba, abaa, baaa; aabb, abab, abba}\}$$

2.1.8. Trito-commutative semiotics: I (α, δ)

$$\text{card}\left(\text{TCS}\left(\text{m, n}\right)\right) = \sum_{k=1}^{M} S\left(n, k\right)\binom{\text{n + m} - 1}{\text{n}}$$

Because of the number series involved,
trito – commutative semiotics might be called *t rito – Mersenne* semiotics.

Example : m = 4, n = 2

$$\sum_{k=1}^{2} S\left(4, k\right)\binom{k + 4 - 1}{k} = \sum_{k=1}^{2} S\left(4, k\right) + \sum_{k=1}^{2}\binom{k + 4 - 1}{k}$$

$$\left(1 + 7\right) + \left(2 + 3\right) = 13.$$

With

IV. $\text{Trito}\left(4, 2\right) = \sum_{k=1}^{2} S\left(4, k\right) = 8$

and

III. $\text{Ind}\left(4, 2\right) = \sum_{k=1}^{2}\binom{k + 4 - 1}{k} = 5$

we get :

I. $\text{Trito}\left(4, 2\right) + \text{Ind}\left(4, 2\right) = \sum_{k=1}^{2} S\left(4, k\right)\binom{k + 4 - 1}{k} = 13.$

I. $= \{$aaaa, bbbb; aaab, aaba, abaa, baaa;
abbb, babb, bbab, bbba; aabb, abab, abba$\}$.

2.1.9. Deutero-partitive semiotics (V, β,ε)

$$\text{card } B^{A} /_{v} = \sum_{k=1}^{M} P\left(m, n\right)\binom{m}{n}$$

whre $n = \text{card } A,\ m = \text{card } B,\ M = \min\left\{\text{card } A, \text{ card } B\right\}$

Example : $m = 2,\ n = 4$

$$\sum_{k=1}^{4} P(m,\ n)\ :$$

$$\sum_{k=1}^{2} P(4,\ k)\ :\ 1\ +2\ =3$$

$$\binom{m}{n}\ :\ \binom{2}{2}\ =1$$

$$\mathbf{V.}\ =\sum_{k=1}^{4} P(m,\ n)\binom{m}{n}\ :3\ +1\ =4$$

$$\mathbf{V.}\ =\{\text{aaaa, bbbb; aaab, aabb}\}.$$

More combinatorial elaborations and results at: *Morphogrammatik*.
http://works.bepress.com/thinkartlab/15/

2.2. Interaction-based classification

2.2.1. Intracontextural bisimilarity

Intracontextural bisimilarity is based on the abstraction over operators on kenogrammatic systems. Depending on the kenogrammatic systems and the set of involved kenogrammatic operators different kind of classifications are to be build. Such bisimilar classifications are not yet involved in cross-contextural constructions and remain inside the realm of contextures of established complexity.

2.2.2. Transcontextural bisimilarity

Transcontextural bisimilarity of classifications is involved into the process of metamorphosis between contextures and their intra-contextural classifications of different kinds.

3. Graphematics of 1 D cellular automata

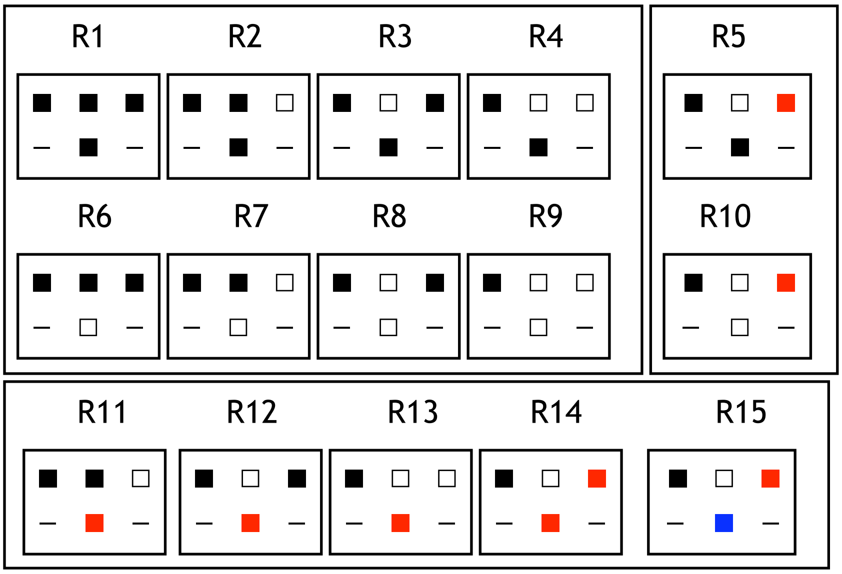
3.1. Graphematic system of cellular automata types

3.1.1. Tritogrammatic CAs

Tritogrammatic CAs are identical to the kenoCAs introduced in previous papers.

$$\text{rules}\left(\text{CA}^{(4,4)}\right) = \sum_{k=1}^{4} \text{Sn2}(4, k) = 1 + 6 + 7 + 1 = 15$$

System of elementary kenomic cellular rules

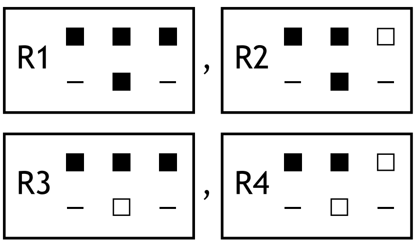


http://memristors.memristics.com/CA-Compositions/Memristive%20Cellular%20Automata%20-Compositions.pdf

3.1.2. Deuterogrammatic CAs

Deuterogrammatic CAs are identical to the deutero-kenoCAs introduced in previous papers.

Deutero – rules for kenoCA$^{(4,2)}$



Deutero kenoCA applications

Nr.l	1	2	3	4	**5**	6	7	8	9	rule = rule1 .4
1	□	□	□	□	■	□	□	□	□	4, 4, 4
2	□	□	□	x	x	x	□	□	□	1, 1, 1, 1, 1
3	□	□	■	■	■	■	■	□	□	4, 4, 1, 1, 1, 4, 4
4	□	x	x	■	■	■	x	x	□	1, 1, 4, 4, 1, 4, 4, 1, 1,
5	■	■	x	x	■	x	x	■	■	stop

Nr.l	1	2	3	4	**5**	6	7	8	9	rule = rule3 .2
1	□	□	□	□	■	□	□	□	□	2, 2, 2
2	□	□	□	■	■	■	□	□	□	2, 2, 3, 2, 2
3	□	□	■	■	x	■	■	□	□	2, 2, 2, 2, 2, 2, 2
4	□	■	■	■	■	■	■	■	□	2, 2, 3, 3, 3, 3, 3, 2, 2
5	■	■	x	x	x	x	x	■	■	stop

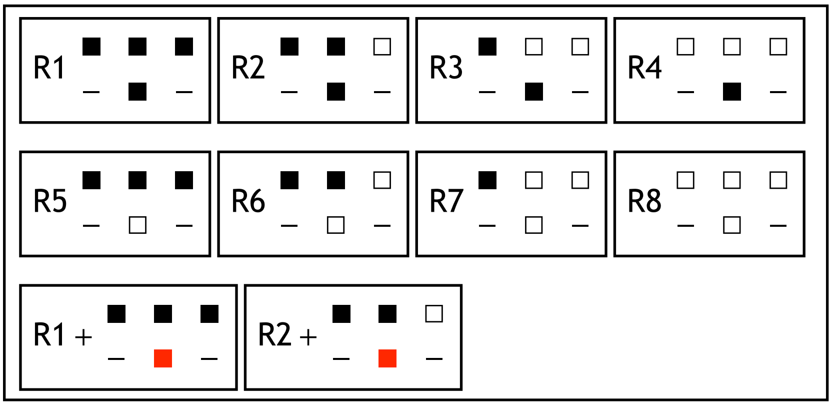
3.1.3. Protogrammatic CAs

Protogrammatic CAs are identical to the proto-kenoCAs introduced in previous papers.

3.1.4. Indicational CAs

Indicational CAs are identical to the indCAs introduced in previous papers.

Enactional rule set for indicational cellular automata indCA



Example

indCA, $r = \{5, 6, 3, 8\}$

Nr.l	1	2	3	4	**5**	6	7	8	9	rule = {5, 6, 3, 8}
1	□	□	□	□	■	□	□	□	□	3, 3, 3
2	□	□	□	■	■	■	□	□	□	3, 6, 5, 6, 3
3	□	□	■	x	x	x	■	□	□	3, 3, 3, 8, 3, 3, 3
4	□	■	■	■	x	■	■	■	□	6, 5, 5, 6, 6, 6, 5, 5, 6
5	x	x	x	x	x	x	x	x	x	stop

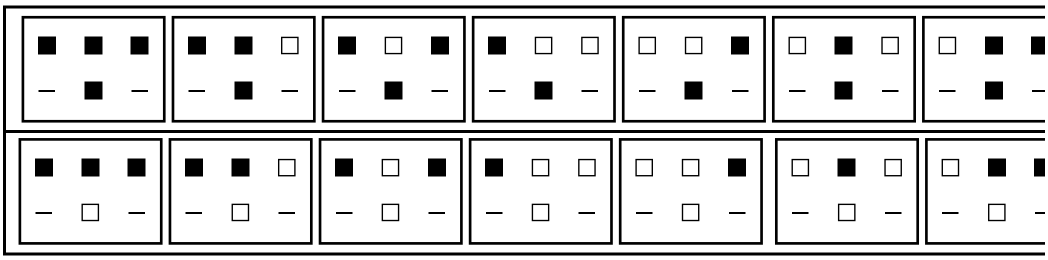
3.1.5. MersenneCAs

$2^3 - 1 = 7$: {aaa, aab, aba, abb, bba, bab, baa},

with (aaa) $=_{eta}$ (bbb).

$\text{MersenneCA}^{(3,2)} = (2^3 - 1) \times P(2, 2) = 7 \times 2 = 14.$

Rules for etaCA$^{(3, 2)}$



The etaCA$^{(3,2)}$ rules are representing $2^3 - 1$ basic rules. Thus the CCA$^{3, 2)}$ rules are represented by just one more, non partitioned rule. Hence, there are two CCA$^{(3, 2)}$ rule sets represented by the etaCA$^{(3,2)}$ basic rule sets.

Example

Nr.l	1	2	3	4	**5**	6	7	8	9	rule = rule1 .2 .10 .4 .12 .6 .7
1	□	□	□	□	■	□	□	□	□	12, 6, 4
2	□	□	□	x	■	■	□	□	□	1, 12, 7, 2, 4
3	□	□	■	x	■	■	■	□	□	12, 6, 12, 4, 1, 9, 4
4	□	x	■	x	■	■	x	■	□	1, 12, 6, 10, 7, 2, 10, 6 , 4
5	■	x	■	x	■	■	x	■	■	stop abstraction : (aaa) $=_{MERS}$ (bbb)

3.1.6. Trito-commutative CAs

Trito – commutative CAs are *a* mix of trito – and commutative structures.

Further informations about memristive cellular automata:
http://memristors.memristics.com/Morphic%20CA/Sketch%20of%20Memristic%20Cellular%20Automata.pdf

3.2. How many mappings are in each equivalence class?

Semiotic cellular automata, ECA or CCA, are produced or introduced in this context as crystallizations of tritogrammatic CAs (kenoCAs).

A tritogram has a number of crystalizations depending on its context, i.e. complexity and complication.

For example, in the context of m=2, n=3, the tritogram [aaa] has just two crystallisations on the semiotic level: (aaa), (bbb), e.g. for the semiotic *identity* and for the *indicational* level.

Hence, a general formula is necessary to determinate the number of crystallizations of tritograms into identities.
With the formulas for the number of trito-, deutero- and proto-abstractions
"we know how many proto-, deutero, and trito-equivalence classes there are in the set B^A of mappings between finite sets A and B.
But how many mappings are in each equivalence class?" (Schadach, p.107, BCL-Report No. 2.2, 1967)

The answers to this question is given by exact formulas which decide how many identitive constellations are restored in the transition from the trito-, deutero- and proto-levels. In other words, how many crystallisations of identive patterns are represented by the graphematic levels of trito-, deutero- and proto-structures. These questions had been answered with two papers by Dieter Schadach (Febr.1967, Aug.1967).
D. J. Schadach, Biomathematik I u. II. (WTB, Bd. 83 u. 87), Berlin/Oxford/Braunschweig 1971. Akademie-Verlag/Pergamon Press/Vieweg + Sohn
http://www.ballonoffconsulting.com/PDF/1987AppendixII.pdf

Trito – level

Theorem 5.
Let the partition of card A = n by the kernel of a mapping $\mu : A \longrightarrow B$ be

$$\pi(n)_{\ker \mu} = (1^{e_1}, 2^{e_2}, \ldots, n^{e_n})$$

and let $[\mu]_t$ denote the trito-equivalence class of μ, i.e., $[\mu]_t \in B^A/_t$. Then

$$\text{card } [\mu]_t = \frac{m!}{(m-k)!}$$

where m = card B and $k = e_1 + e_2 + \ldots + e_n = \text{card } {}^A/_{\ker \mu}$.

The number of mappings in the *trito* – class is given by card $[\mu]_t$:

$$\text{card}\left[\mu\right]_t = \frac{m!}{(m-k)!}$$

Example

$$m = k = 3,\ \frac{3!}{(3-3)!} = 3! = 6$$

$$\left[abc\right]_{trito} \Rightarrow \left\{(abc), (acb), (bca), (bac), (cba), (cab)\right\}_{ident}.$$

Hence, the tritogram [abc] has 6 identitive representations because all semiotic permutations of the tritogram [abc] are kenogrammatically equivalent.

For $m = 3$,

$k = 2$ there are 6 equivalent representation of the tritogram $\left[abb\right]$:

$$\frac{3!}{(3-2)!} = 3! = 6$$

$$\left[abb\right]_{trito} \Rightarrow \left\{(abb), (acc), (baa), (bcc), (caa), (cbb)\right\}_{ident}.$$

Schadach's example for trito – class card $\left[\mu\right]_{trito}$

$$\sum_{k=1}^{M} \frac{m!}{(m-k)!},\ n = 6,\ m = 5,\ M = \min\{n, m\} = 5,\ k = 1, 2, \ldots, 5.$$

$$m = 5,\ k = 1,\ \frac{5!}{(5-1)!} = \frac{5!}{4!} = 5$$

$$m = 5,\ k = 2,\ \frac{5!}{(5-2)!} = \frac{5!}{3!} = 20$$

$$m = 5,\ k = 3,\ \frac{5!}{(5-3)!} = \frac{5!}{2!} = 60$$

$$m = 5,\ k = 4,\ \frac{5!}{(5-4)!} = \frac{5!}{1!} = 120$$

$$m = 5,\ k = 5,\ \frac{5!}{(5-5)!} = \frac{5!}{0!} = 120$$

Hence,

$$\sum_{k=1}^{5} \frac{5!}{(5-k)!} = 325.$$

Application

$$m = 9,\ k = 4,\ \frac{9!}{(9-4)!} = \frac{9!}{5!} = 3024$$

$$[\text{abbaabccd}]_{trito} = \{(\text{abbaabccd}),\ \ldots,\ (\text{dccddcbba})\}_{ident}$$

A trito key [abbabccd] in a security system represents 3024 different semiotic keys for the trito-key out of the 9^4= 6561 possible different identitive semiotic keys.

http://www.thinkartlab.com/pkl/media/Web_Mobility/Web_Mobility.pdf

Deutero-level

Theorem 6.

Let the partition of card A = n by the kernel of a mapping $\mu: A \longrightarrow B$ be

$$\pi(n)_{ker\ \mu} = (1^{e_1}, 2^{e_2}, \ldots, n^{e_n})$$

and let $[\mu]_d$ denote the deutero-equivalence class of μ, i.e., $[\mu]_d \in D^A/_d$. Then

$$\text{card}\ [\mu]_d = \frac{n!}{(1!)^{e_1}(2!)^{e_2} \ldots (n!)^{e_n}} \binom{m}{k} \frac{k!}{e_1!\, e_2! \ldots e_n!}$$

where m = card B and $k = e_1 + e_2 + \ldots + e_n = \text{card}\ A/_{ker\ \mu}$.

The number of mappings in the *deutero*-class is given by card $[\mu]_d$:

$$\text{card}[\mu]_d = \frac{n!}{(1!)^{e_1}(2!)^{e_2} \ldots (n!)^{e_n}} \binom{m}{k} \frac{k!}{e_1!\, e_2! \ldots e_n!}$$

Proto – level

The number of mappings in the *proto* – class is given by card $[\mu]_P$:

$$\mathrm{card}[\mu]_P = \sum_{i=1}^{P(n,k)} \frac{n!}{(1!)^{e_{i1}}(2!)^{e_{i2}}\dots(n!)^{e_{in}}}\binom{m}{k}\frac{k!}{e_{i1}!\,e_{i2}!\dots e_{in}!}$$

Example.

Let be $A = \{a_1, a_2, \dots a_6\}$ and $B = \{b_1, b_2, \dots b_5\}$. Then card $A = n = 6$;
card $B = m = 5$; card $B^A = m^n = 15625$; $M = \min\{n,m\} = 5$; $k = 1, 2, \dots, 5$.

$\pi(n)_{\ker\mu}$	k	P(n,k)	S(n,k)	card $[\mu]_P$	card $[\mu]_d$	card $[\mu]_t$
6^1	1	1	1	5	5	5
$1^1, 5^1$ $2^1, 4^1$ 3^2	2	3	31	620	120 300 200	20
$1^2, 4^1$ $1^1, 2^1, 3^1$ 2^3	3	3	90	5400	900 3600 900	60
$1^3, 3^1$ $1^2, 2^2$	4	2	65	7800	2400 5400	120
$1^4, 2^1$	5	1	15	1800	1800	120

3.2.1. Semiotic cellular automata representations

Semiotic cellular automata are the common type of CAs, based on the identity principle of the use of their signs and methods.

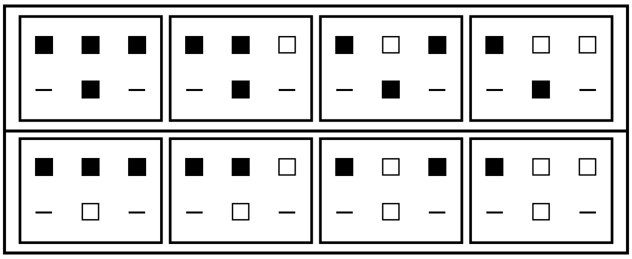
The question is: *How are semiotic CAs represented by kenogrammatic CAs of the trito-structure?*

The answer is given by the combinatorial considerations regarding the translation or mapping of trito-structures into identity structures.

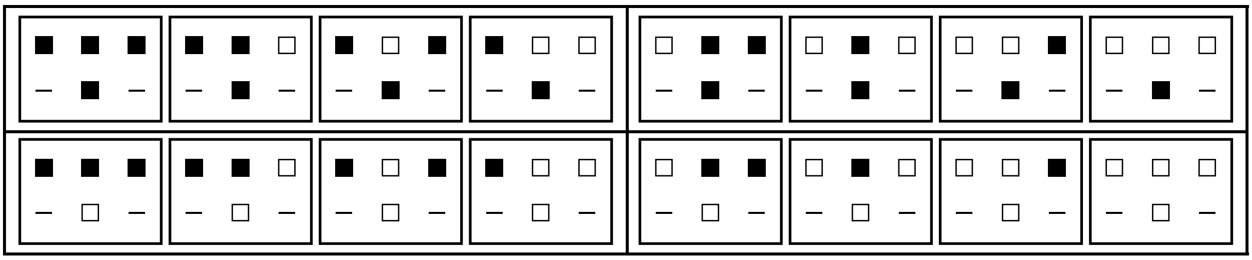
This translation is fairly trivial if classical 1 D CAs are considered. For their binary complexity with m=2 and complication n with n=3, the *rule range* is 2^8=256.

Hence the trito-structure tritoCA$^{(3,\ 2)}$ has a set of the *single local* elementary rules of
Stirling2(2, 3) x P(2, 2): 4 x 2 = 8, with a *rule space* of
$\sum_{k=1}^{2}$ Sn2(4, k) x P(2, k) = 16.

Rules for tritoCA$^{(3,\ 2)}$

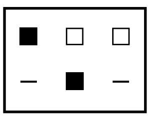


Semiotic rulesCA$^{(3,\ 2)}$

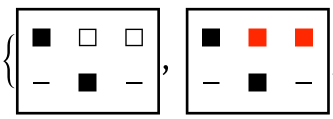


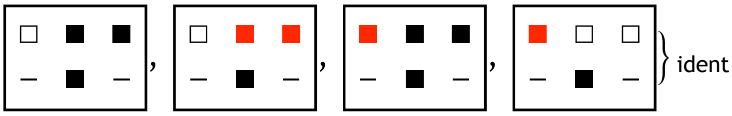
$$m = k = 3$$

$$[\text{abb}]_{\text{trito}} \Rightarrow \{(\text{abb}),\ (\text{acc}),\ (\text{baa}),\ (\text{bcc}),\ (\text{caa}),\ (\text{cbb})\}_{\text{ident}}.$$



trito $\Rightarrow$

{  ,



This is *work in progress* and needs much more mathematical elaboration. It is, again, a hint in the right direction - not more but not less.