Gödel's Games within Palindromes

Morphic Palindromicity as a Measure for Self-Reference

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Abstract

The paper *"Gödel Games: Cloning Gödel's Proofs"* started a polycontextural dissemination of the *"Beautified Gödel Proofs"* of Hehner. This paper takes a morphogrammatic turn to such a dissemination in emphasizing the distributive possibilities of the modi of repetition in morphic palindromes. Palindromicity of morphograms is a precision and concretization of the philosophical construct of iterability as it was applied for a distribution of Gödel's theorems.
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1. Self-reference without reference?

1.1. Reflections on "Gödel's Games"

1.1.1. Polycontextural modeling

> Again, """In a slogan: "Quotes don't know their mates.""
> George Boolos, Logic, Logic, and Logic"

The paper *"Gödel Games: Cloning Gödel's Proofs"* started a dissemination of the *"Beautified Gödel Proofs"* of Hehner. The emphasis was on a complex distribution, *reflectional* and *interactional*, of the mechanism of "self-quotation", ruled by the interplay of *quotation* and *interpretation*, as the device of the Gödel construction.

Hence, distribution as a modus of repetition was conceived as *"iterative"* and as *"accretive"* and realized in a grid of contextures.

As a further *concretization* of the idea of quotation and dissemination (distribution and mediation), an application of morphogrammatic *palindromic* constructions shall be risked. Because of the lack of any information about the *internal* differences of the disseminated construction of the polycontextural approach, the chances to fill this gap by a 'palindromic'

interpretation of iteration shall be taken.

A Turing machine TM M applied on its own 'description' [M] leads by diagonalization/normalization to the desired self-referential machine M[M].

M : M $\longrightarrow$ [M $\rightleftarrows$ M]

Gödel's and Smullyan's construction are presuming semiotic identity between the 'active' definition of the TM M and the 'passive', i.e. the quotation [M] of the active TM M. The relationship between M and [M] is hierarchical, and is modeled as a relationship of program (processor) and data, or 'operator' and 'operand'.

Therefore, the texts of M and [M] have to be semiotically (symbolically) identical. The quotation [M] of M is operationally a 'mirrored' and 'replicated' text of M.

But this corresponds abstractly the definition of a *symmetric palindrome*. A mirror-image is the inverse or dual of the mirrored original. In this scenario, the original comes first, the mirrored image second and in reverse order, both together are involved into the relationship or process of self-similar and self-referential mappings and interactions. This still holds if the replication of the original is mirrored as an iteration of the same identitive structure.

Certainly, that's not the standard logical and linguistic definition of a quotation as Smullyan's example shows directly: *John is reading "John is reading"*. Here, the quotation is a literal replication of the first occurrence of "*John is reading*" by the quote "*John is reading*". This surface-structural approach says nothing about the morphic structure of the construction: the composition of the first with the replication of the first as the second in the mode of identity.

To replicate in the mode of identity presupposes a decision in favor for identity, i.e. for equality, in contrast to the possibility of *equivalence*, *similarity* or *bisimilarity* - and others.

Folowing the insights of the morphogrammatic "T*rompe-l'œils* of Semiospheres" of semiotic configurations we get, e.g. for X = [1,2,3,4], the morphogrammatic result, X"X":

- ispalindrome [1,2,3,4,1,2,3,4];
val it = true : bool

And the semiotic result:

- palindrome [1,2,3,4,1,2,3,4];
val it = false : bool

But not all mirrors are offering a symmetric replication of the 'original' text. Asymmetric textures with symmetric functionality are the morphogrammatic subversions, enabled and played by morphic palindromes.

Hence, the Gödelian misery (disapointment) of the limitation theorems for Kurt Gödel is inherited by its complicity with the historical concepts of semiotic palindromes and its symmetry.

Historically, Kurt Gödel met Gotthard Gunther a few years to early to get some valuable hints from Gunther to overcome the negativity of his own results. (Charles Parsons) There was probably a mismatch of interests too. Gunther wanted some technical help for his attempts to formalize his reflectional logic (Stellenwertlgik). Gödel assumed to get some hints for conceptual and philosophical inspirations. Both lacked a mediator to help each other.

The proposed dissemination of "Gödel" in the paper *"Gödel Games: Cloning Gödel's Proofs"* wasn't yet contemplating about the difference of 'symmetrical' and 'asymmetrical' formulations of a Gödel sentence, here in the context of a Turing machine TM and its program M.

What happens if the quotation quotes creatively a different inscription that is nevertheless palindromically equivalent to its origin?

What happens if the process of cloning is accretively, and not iteratively repeating its iterated origin?

Thus, the Gödel theorems are based on symbolic palindromes. Post-Gödelian theorems are forced by morphic palindromes.

Why would we need self-referentiality, and all its derivates, like self-applications, etc.?

This *subversion* applied on the definition of the Halting problem has now to consider the double character of self-application, the symmetric and the asymmetric, or as it was conceived before, the *iterative* and the *accretive* modi of iterability.

Things might be confusing! A first confusion is cleared by Sipser's advice: *"Don't be confused by the idea of running a machine on its own description! That is similar to running a program with itself as input, something that does occasionally occur in practice."* (Sipser, 1997, p.165)

Obviously, all philosophical and logical considerations and problems with self-reference are generously bracket out by Sipser's advice.

The other confusion is harder to disperse. As shown in the paper *"Morphosphere(s)"* palindromes might be asymmetric in the framework of

morphogrammatics, i.e. in the paradigm of morphospheres. This possibility of a simultaneity of semiotic asymmetry and a morphic symmetry is the new challenge to the theory of computation.

With the palindromic subversion, the idea of disseminating Gödel's proof over a polycontextural grid gets a radical concretization, and a possibility of a much more direct elaboration.

Hence, 'measures' of undecidability are accessible to intriguing concretions, enabled by the complexity of asymmetric palindromes. Asymmetric palindromes functions as asymmetric *quotations* and *iterations* or repetitions and replications.

The formula <X"X"> always presupposes that X≡X, i.e., that X and the quoted (repeated) X in the quotation "X" are equal. It doesn't say that the unquoted X and the quoted X, i.e. "X", are equal. One is on a first-order, the other on a second-order level. One occurs first, the other second, hence, the second is a repetition of the first. But what is repeated is X and X is equal as X at both positions of occurrence.

Therefore, the nice text-book presentations of Gödel's theorems, the self-applications of programs and their *diagonalizations* (Cantor, Tarski, Gödel) and *normalizations* (Smullyan) are appearing as a tiny special case of *identity* strategies in the general framework of graphematic scriptures.

This sounds trivial but if iteration *alters* then the alteration needs to be characterized.

> *"The defined theory* "allow us to replace something with its equal" *but it is not able to disallow a distributed substitution because equality of terms is defined in the theory only* "up to isomorphism". *Such theories are identifying the terms* "equal" *with* "same". *The polycontextural approach offers a different option to the difference of equality and sameness. Equality in this sense is an intra-contextural term but sameness is a trans-contextural term. Because substitution is generally defined in a theory only up to isomorphism we always have the possibility to interpret the action also in a trans-contextural way. As long as the definitions of the theory are not disallowing this way to use substitution there is always some degree of freedom to interpret the terms in another similar theory."*
> http://www.thinkartlab.com/pkl/lola/Godel_Games/Godel_Games.htm

To quote something as something else that still is the same:
Symbolic quotation
To quote something as itself, i.e. in the mode of the is-abstraction as: "X

as X is X".
Morphic quotation
To quote something in the mode of the morphic as-abstraction as: "X as Y is Z".
Metamorphic quotation
To quote something in the mode of the metamorphic as-abstraction as: "X as Y is U as V".

Hence, *"Self-reference without reference"* is the slogan for the fact that morphogrammatic referencing is evoking its reference in the process of referring to its own reference.

1.2. Smullyan's Norm: *N "N"*

There are many attempts and wordings to introduce the scenario of self-reference in formal languages. It is not my aim to add one more. Therefore I shall quote Raymond Smullyan's quotations about the process of quotation, its explanation and formalization as it is presented on a semi-formal level.

> ### Normation and diagonalization
> *"By the norm of an expression we shall mean the expression followed by its own **quotation**. For example, consider the following expression.*
>
> *(1) John is reading*
> *The norm of (1) is the following.*
> *(2) John is reading "John is reading"*
>
> *The sentence (2) is not self-referential; it doesn't assert that John is reading (2); it asserts that John is reading (1).*
> *But now consider the following.*
>
> *(3) John is reading the norm of*
> *Its norm is the following sentence.*
> *(4) John is reading the norm of "John is reading the norm of"*
>
> *Sentence (4) asserts that John is reading the norm of (3), but the norm of (3) is (4) itself. And so (4) asserts that John is reading (4). Thus (4) is self-referential.*
>
> *Let us look at an abbreviated version. Again, we abbreviate "John is reading" by the letter "J." And we shall use the letter "N" to abbreviate "the norm of." Then (3) and (4) assume the following symbolic forms.*
>
> *(3)' JN*
> *(4)' JN"JN"*
>
> *(4)' asserts that John is reading the norm of (3)', but the norm of (3)' is*

(4)' itself. Thus (4)' asserts that John is reading (4)'.

Alternatively, using our notion of norm, the following is a version of the paradox.
The set of false sentences contains the norm of "the set of false sentences contains the norm of"

Using diagonalization instead of normalization, the paradox would be stated thus.

The set of false sentences contains the diagonalization of
"the set of false sentences contains the diagonalization of x"

*Let us note in passing that, using **diagonalization**, the following is an expression that designates itself: the diagonalization of "the diagonalization of x." Or symbolically: D"Dx". Using **normalization**, the following expression designates itself: the norm of "the norm of." Or symbolically: N"N"." (Smullyan, 1994, p. 4/5)*

Repetition
"A conceptually simpler operation than association, which also accomplishes self-reference, is that of repetition, where by the repeat of an expression X is meant the expression XX, i.e., X followed by itself.

Let us use "R" to abbreviate "the repeat of." Then a sentence that asserts that John is reading it is JR*JR*. It asserts that John is reading the repeat of JR*, which is the very sentence JR*JR*. (The sentence JR*JR won't work; it asserts that John is reading JRJR, not that John is reading JR*JR.)"

Raymond M. Smulyan, Diagonalization and Self-Reference, 1994
http://bookos.org/g/Raymond%20M.%20Smullyan
http://www.thinkartlab.com/pkl/lola/Godel_Games/Godel_Games.htm
http://www.thinkartlab.com/pkl/lola/Godel_Games-short.pdf

The general strategy
"Some of mathematicians, for example George Polya, argued that deductions is impotent without analogy. What I want to show is that there exists a direct line leading from Cantor's diagonal argument to constructions that underlies of the proofs of several important theorems of the mathematical logic (in particular, Church's theorem concerning the undecidability of formal arithmetic, Gödel's theorem concerning the incopleteness of formal arithmetic, Tarski's theorem concerning truth, and Turing's theorem concerning the Halting Problem), and that the line could be described as an analogical mapping. In other words, Cantor's diagonal argument and the proofs of the limitative theorems are

structurally the same."
Zbigniew Tworak, ANALOGY AND DIAGONAL ARGUMENT
www.logika.umk.pl/llp/151/151-3.pdf

2. Palindromic analysis of morphic quotation

2.1. Examples of palindromic quotations

2.1.1. General motivation

Thesis:

Morphic self-referentiality is palindromic. Producing finite self-quotational palindromes.

Indicational self-referentiality is a re-entry of the form into the form. Producing an infinite re-entry form.

Symbolic self-reference of a sentence is based on its self-quotation caught by its norm or diagonalization. Producing infinite iterations and logical contradictions.

The *palindromic* approach to quotation as repetition, replication or reflection gives a closer connection between the formula and its quotation than a free, i.e. a structurally not mediated connection to the original formula in a repetition of the formula.

"By the norm *of an expression we shall mean the expression followed by its own* quotation." (Smullyan)

All depends now on the understanding of the term "own". What does "own" mean in the relation to its own sentence. Who owns this sentence that has a relationship to its own?

Obviously, the ownership of both, the sentence and its own quotation is a property of the identity logic and semiotics of logocentrism.

If an 'expression' should be quoted (by another expression) it necessarily has to be the identically same sentence of the quotation that has to be quoted. And not accidentally another possible sentence. Identity secures the truthfulness of the relationship between the original and its replication as a quotation. This is well codified by: $\lceil\lceil M\rceil\rceil = M$. Hence, identity is the measure or criterion of the success of the interplay of the original and the replication of the original on a different linguistic and logical level. It guarantees the successful bridging of the different levels.

Morphograms are not sentences, thereafter there is no ownership by logocentric self-reference and logic possible.

In this sense, it makes, without apophantic reference, sense, to refer to what has no reference, the iterability of inscription, technically realized by morphic palindromes.

If morphograms are not linguistic entities or processes like sentences how could a quotation be realized?

A first attempt to tackle this intriguing question might be achieved by the focus on the kind of *iterability* between the 'original' and its 'iteration' or 'replication'. Independent of cognitive and linguistic levels of thematizations, the statement of the original and the repetition of it on a different level as a quote, the common or deep-structure of it is the action of iteration. Here, restricted on a kind of a linear order of the first and the second.

With morphogram X = [1,2,2,3] as the stated 'original' and "X" = [2,1,1,3] as the quotation of the original "itself" as "another" [2,1,1,3] with [2,1,1,3] $=_{MG}$ [1,2,2,3], the *composed* morphogram X"X" = [1,2,2,3,2,1,1,3] is introduced as a 'quotational' composition. But is it a palindrome? The composed morphogram X"X" is a quotation if it is a palindrome.

Also X and "X" of the example are morphogrammatically equivalent, the composition X"X" = [1,2,2,3,2,1,1,3] is *not* a morphic palindrome. Albeit the fact that the morphogram [2,1,1,3] is a sort of an *iteration* or even an *accretion* of the morphogrammatically equivalent morphogram X = [1,2,2,3], the composed morphogram fails the criterion of palindromicity.

In contrast, the composition X"X" with X = [1,2,2,3] and "X" = [3,1,1,2], X"X" = [1,2,2,3,3,1,1,2] is a palindrome, and is therefore qualified as a morphic quotation. The composition <X"X"> might be written as [[1,2,2,3],[3,1,1,2]] to emphasize its two components.

Hence, the new 'ownership' of the 'reference' to its 'own' inscription is in the ownership and control of what is enabling morphic palindromes. Morphic palindromes own the rules of morphic quotations.

2.1.2. Self-referentiality of N"N" and [N"N"]

How is morphic self-referentiality constructed?

Smullyan:
"(3) John is reading the *norm* of x
Its norm is the following sentence.
(4) John is reading the *norm* of x="John is reading the *norm* of x"
Sentence (4) asserts that John is reading the norm of (3), but the norm of (3) is (4) itself. And so (4) asserts that John is reading (4). Thus (4) is **self-referential**."

Transcription

(((John is reading the norm=John is reading the norm)=John is reading the norm of))... of "John is reading the norm of"...)))

N"N" $\Rightarrow$ N"N"N"" $\Rightarrow$ N"N"N"N"""" $\Rightarrow$ ...

Morphogrammatic model

The same is different

Iteration alters. Hence, the iterative procedure is producing a chain of same and different quotations for N"N" depending on the modi of iterativity, replication, reversion and accretion. All have to accept the conditions of palindromicity to realize a morphic quotation.

Hence, morphic self-referentiality, using different modi of iteratiity, is not circular but is creating labyrinthine journeys that might contain circles too.

The different is the same

Different morphograms are the same at different places. Quotation is depending on (reflectional, object-, meta-language) places in a contextural grid.

Hence, morphograms N and "N" are morphogrammatically equivalent (similar): $[N] =_{MG} ["N"]$.

Is there a difference between ["X"] and "[X]"?

Thus, morphic self-referentiality is not contradictory. The (negational, reflectional) differences are collapsing in the negation-invariant morphogram.

What are the analogies of contradictions for morphogrammatic systems?

2.1.3. Quotations as palindromes

Also the two parts, [1,2,2,3] and [2,1,1,3] of the composition X"X", [1,2,2,3,2,1,1,3], are morphogrammatically equivalent, $[X] =_{MG} ["X"]$, the composition is not a palindrome.

ispalindrome X"X"?
- ispalindrome [1,2,2,3, 2,1,1,3];
val it = false : bool

On the other hand, other morphic quotations of X are compatible with palindromes.

X"X" with X = [1,2,2,3] and "X" = {[3,1,1,2], [3,2,2,1], [4,2,2,1], [4,5,5,6]} are morphic palindromes.

- ispalindrome [1,2,2,3, 3,1,1,2];
val it = true : bool
- ispalindrome [1,2,2,3, 3,2,2,1];
val it = true : bool

```
- ispalindrome [1,2,2,3, 4,2,2,1];
val it = true : bool
- ispalindrome [1,2,2,3,4,5,5,6];
val it = true : bool
Replication
- ispalindrome[1,2,2,3,1,2,2,3];
val it = true : bool
```

Again, the *redundant* pattern [1,2,2,3,4,6,6,7] with [1,2,2,3] and [4,6,6,7] is not accepted as a palindrome:

```
- ispalindrome [1,2,2,3, 4,6,6,7];
val it = false : bool
- tnf[1,2,2,3,4,6,6,7];
val it = [1,2,2,3,4,5,5,6] : int list
```

Following the classic definition for a palindrome as < X"X">, with "X" = inverse(X), the morphogrammatic equivalent shall be:
        [X]"[X]" is a palindrome if "[X]" = kref[X].

Examples
```
X = [1,2,2] is not palindrome
 - kref[1,2,2];
val it = [1,1,2] : int list
X, krefX:
- ispalindrome[1,2,2,1,1,2];
val it = true : bool
```
```
X = [1,2,2,3] is a palindrome
- kref[1,2,2,3];
val it = [1,2,2,3] : int list
- ispalindrome[1,2,2,3,1,2,2,3];
val it = true : bool
```
```
X= [1,1,2,3] is not a palindrome
- kref[1,1,2,3];
val it = [1,2,3,3] : int list
- ispalindrome[1,1,2,3,1,2,3,3];
val it = true : bool
```
```
- rev[1,2,2,3];
val it = [3,2,2,1] : int list
- ispalindrome[1,2,2,3,3,2,2,1];
val it = true : bool
```

2.1.4. Exemplary elaboration for *Tcontexture 8*

In accordance to the concept of morphograms, domains of morphograms are finite and numerically characterized by the sum of the Stirling numbers of the second kind. This kind of finiteness allows to study the whole domains.

For the trito-structure of graphematical systems, such domains are formally defined as *Tcontextures n*.

The set of palindromes is filtered out from Tcontexture 8 by the ML-function "*List.filter*". The palindromes of *Tcontextures* are 'found' here by 'search' and not by construction.

Tcontexture 8
- length(Tcontexture 8);
val it = 4140 : int

List.filter ispalindrome "Tcontexture 8":
val it =
 [[1,1,1,1,1,1,1,1],[1,1,1,1,2,2,2,2],[1,1,1,2,1,2,2,2],[1,1,2,1,2,1,2,2],
  [1,1,2,2,1,1,2,2],[1,1,2,2,2,2,1,1],[1,2,1,1,2,2,1,2],[1,2,1,2,1,2,1,2],
  [1,2,1,2,2,1,2,1],[1,2,2,1,1,2,2,1],[1,2,2,1,2,1,1,2],[1,2,2,2,1,1,1,2],
  [1,1,1,2,2,1,1,1],[1,1,2,1,1,2,1,1],[1,2,1,1,1,1,2,1],[1,2,2,2,2,2,2,1],
  [1,1,1,2,2,3,3,3],[1,1,2,3,3,1,2,2],[1,1,2,1,3,2,3,3],[1,1,2,3,1,2,3,3],
  [1,2,1,3,3,2,1,2],**[1,2,2,3,3,1,1,2]**,[1,2,3,1,2,3,1,2],[1,2,3,2,1,3,1,2],
  [1,2,1,1,3,3,2,3],[1,2,1,3,1,3,2,3],[1,2,3,1,3,1,2,3],[1,2,3,3,1,1,2,3],
  [1,2,2,2,3,3,3,1],**[1,2,2,3,2,3,3,1]**,[1,2,3,2,3,2,3,1],[1,2,3,3,2,2,3,1],
  [1,1,2,2,3,3,1,1],[1,1,2,3,2,3,1,1],[1,1,2,3,3,2,1,1],[1,2,1,2,3,1,3,1],
  [1,2,1,3,2,1,3,1],[1,2,1,3,3,1,2,1],[1,2,2,1,1,3,3,1],[1,2,3,1,1,2,3,1],
  [1,2,3,1,1,3,2,1],[1,2,2,1,3,2,2,3],**[1,2,2,3,1,2,2,3]**,**[1,2,2,3,3,2,2,1]**,
  [1,2,1,2,2,3,2,3],[1,2,3,2,2,1,2,3],[1,2,3,2,2,3,2,1],[1,1,2,2,2,2,3,3],
  [1,2,3,3,3,3,1,2],[1,2,3,3,3,3,2,1],[1,1,1,2,3,1,1,1],[1,1,2,1,1,3,1,1],
  [1,2,1,1,1,1,3,1],[1,2,2,2,2,2,2,3],[1,1,2,2,3,3,4,4],[1,1,2,3,2,3,4,4],
  [1,1,2,3,3,2,4,4],[1,2,1,2,3,4,3,4],[1,2,1,3,2,4,3,4],[1,2,1,3,3,4,2,4],
  [1,2,2,1,3,4,4,3],**[1,2,2,3,1,4,4,3]**,[1,2,2,3,3,4,4,1],[1,2,3,1,4,2,3,4],
  [1,2,3,1,4,3,2,4],[1,2,3,2,4,1,4,3],[1,2,3,2,4,3,4,1],[1,2,3,3,4,4,1,2],
  [1,2,3,3,4,4,2,1],[1,2,3,4,1,2,3,4],[1,2,3,4,1,3,2,4],[1,2,3,4,2,1,4,3],
  [1,2,3,4,2,3,4,1],[1,2,3,4,3,4,1,2],[1,2,3,4,3,4,2,1],[1,2,3,4,4,1,2,3],
  [1,2,3,4,4,2,3,1],[1,2,3,4,4,3,1,2],[1,2,3,4,4,3,2,1],[1,1,1,2,3,4,4,4],
  [1,1,2,3,4,1,2,2],[1,1,2,1,3,4,3,3],[1,1,2,3,1,4,3,3],[1,2,1,3,4,2,1,2],
  [1,2,2,3,4,1,1,2],[1,2,3,1,2,4,1,2],[1,2,3,2,1,4,1,2],[1,2,1,1,3,3,4,3],
  [1,2,1,3,1,3,4,3],[1,2,3,1,3,1,4,3],[1,2,3,3,1,1,4,3],[1,2,2,2,3,3,3,4],
  [1,2,2,3,2,3,3,4],[1,2,3,2,3,2,3,4],[1,2,3,3,2,2,3,4],[1,1,2,3,4,2,1,1],
  [1,1,2,3,3,4,1,1],[1,2,1,3,4,1,2,1],[1,2,3,1,1,4,2,1],[1,2,1,3,3,1,4,1],
  [1,2,3,1,1,3,4,1],**[1,2,2,3,4,2,2,1]**,[1,2,3,2,2,4,2,1],[1,2,3,3,3,3,4,1],
  [1,2,2,3,3,2,2,4],[1,2,3,2,2,3,2,4],[1,2,3,3,3,3,2,4],[1,1,2,3,4,2,5,5],
  [1,1,2,3,3,4,5,5],[1,2,1,3,4,5,2,5],**[1,2,2,3,4,5,5,1]**,[1,2,3,4,5,1,2,3],
  [1,2,3,4,5,2,3,1],[1,2,3,4,5,3,1,2],[1,2,3,4,5,3,2,1],[1,2,3,1,4,5,2,4],
  [1,2,3,2,4,5,4,1],[1,2,3,4,1,5,2,4],[1,2,3,4,2,5,4,1],[1,2,3,4,4,5,1,2],
  [1,2,3,4,4,5,2,1],[1,2,1,3,3,4,5,4],[1,2,3,1,4,3,5,4],[1,2,3,3,4,4,5,1],
  [1,2,3,4,1,3,5,4],[1,2,3,4,3,4,5,1],[1,2,3,4,4,1,5,3],[1,2,3,4,4,3,5,1],
  [1,2,2,3,3,4,4,5],[1,2,3,2,4,3,4,5],[1,2,3,3,4,4,2,5],[1,2,3,4,2,3,4,5],

[1,2,3,4,3,4,2,5],[1,2,3,4,4,2,3,5],[1,2,3,4,4,3,2,5],[1,1,2,3,4,5,1,1],
[1,2,1,3,4,1,5,1],[1,2,3,1,1,4,5,1],**[1,2,2,3,4,2,2,5]**,[1,2,3,2,2,4,2,5],
[1,2,3,3,3,3,4,5],[1,1,2,3,4,5,6,6],[1,2,3,4,5,6,1,2],[1,2,3,4,5,6,2,1],
[1,2,1,3,4,5,6,5],[1,2,3,4,5,1,6,3],[1,2,3,4,5,3,6,1],[1,2,3,1,4,5,6,4],
[1,2,3,4,1,5,6,4],[1,2,3,4,4,5,6,1],**[1,2,2,3,4,5,5,6]**,[1,2,3,4,5,2,3,6],
[1,2,3,4,5,3,2,6],[1,2,3,2,4,5,4,6],[1,2,3,4,2,5,4,6],[1,2,3,4,4,5,2,6],
[1,2,3,3,4,4,5,6],[1,2,3,4,3,4,5,6],[1,2,3,4,4,3,5,6],[1,2,3,4,5,6,7,1],
[1,2,3,4,5,6,2,7],[1,2,3,4,5,3,6,7],[1,2,3,4,4,5,6,7],[1,2,3,4,5,6,7,8]]
 : int list list
- length it;
val it = 164 : int

Symmetric palindromes from Tcontexture 8:
val it =
 [[1,1,1,1,1,1,1,1],[1,1,2,2,2,2,1,1],[1,2,1,2,2,1,2,1],[1,2,2,1,1,2,2,1],
 [1,1,1,2,2,1,1,1],[1,1,2,1,1,2,1,1],[1,2,1,1,1,1,2,1],[1,2,2,2,2,2,2,1],
 [1,1,2,3,3,2,1,1],[1,2,1,3,3,1,2,1],[1,2,3,1,1,3,2,1],**[1,2,2,3,3,2,2,1]**,
 [1,2,3,2,2,3,2,1],[1,2,3,3,3,3,2,1],[1,2,3,4,4,3,2,1]] : int list list

The *emphazised* palindromes, e.g. **[1,2,2,3,4,5,5,1],** are the palindromes
with the presuposed header **[1,2,2,3].**

The examples for the 'quotational' palindromes are therefore defined by:
    List.filter ispalindrome "Tcontexture 8" modulo head= "[1,2,2,3]".

- Tcontexture 4;
val it =
 [[1,1,1,1],[1,1,2,2],[1,2,1,2],[1,2,2,1],[1,1,1,2],[1,1,2,1],[1,2,1,1],
 [1,2,2,2],[1,1,2,3],[1,2,1,3],[1,2,3,1],[1,2,2,3],[1,2,3,2],[1,2,3,3], [1,2,3,4]] :
int list list
- List.filter ispalindrome "Tcontexture 4"
val it =
 [[1,1,1,1],[1,1,2,2],[1,2,1,2],[1,2,2,1],[1,2,3,1],[1,2,2,3],[1,2,3,4]] : int list list
For head [1,2]:
[1,2,1,2],[1,2,2,1],[1,2,3,1],[1,2,2,3],[1,2,3,4].

Modi
repetition: [1,2,1,2],
inversion: [1,2,2,1],
accretion: [1,2,3,1],[1,2,2,3],[1,2,3,4].

Odd morphograms
- Tcontexture 3;
val it = [[1,1,1],[1,1,2],[1,2,1],[1,2,2],[1,2,3]] : int list list
- List.filter ispalindrome [[1,1,1],[1,1,2],[1,2,1],[1,2,2],[1,2,3]];
val it = [[1,1,1],[1,2,1],[1,2,3]] : int list list

Modi for head [1,2]:
repetition: [1,2,1],

inversion: -
accretion: [1,2,3].

Tcontexture 5
- List.filter ispalindrome "Tcontexture 5"
val it =
  [[1,1,1,1,1],[1,2,1,2,1],[1,2,2,2,1],[1,1,2,1,1],[1,1,2,3,3],[1,2,3,1,2],
   [1,2,3,2,1],[1,2,1,3,1],[1,2,2,2,3],[1,2,3,4,1],[1,2,3,2,4],[1,2,3,4,5]]
  : int list list

Partial quotations
Modi for head [1,2]:
repetition: [1,2,1,2,1],
inversion: [1,2,2,2,1]
accretion:
[1,2,3,1,2],[1,2,3,2,1],[1,2,1,3,1],[1,2,2,2,3],[1,2,3,4,1],[1,2,3,2,4],[1,2,3,4,5].

Modi for head [1,2,1]:
repetition: –
inversion: [1,2,1,2,1], [1,2,2,2,1],
accretion:
[1,2,3,1,2],[1,2,3,2,1],[1,2,1,3,1],[1,2,2,2,3],[1,2,3,4,1],[1,2,3,2,4],[1,2,3,4,5].

Quotations with a 'neutral' midpoint for palindromes(5)
head: [1,2], midpoint[x] of length=1
repetition: [1,2,**3**,1,2],
inversion: [1,2,**1**,2,1],[1,2,**2**,2,1],[1,2,**3**,2,1],
accretion: [1,2,**1**,3,1],[1,2,**2**,2,3],[1,2,**3**,4,1],[1,2,**3**,2,4],[1,2,**3**,4,5].

2.1.5. Different types of quotations

Modi of quotation and morphic palindromicity. Palindromicity is guaranteeing the adequateness of the modi of morphic quotations. Palindromes are preserving the quotational relatedness between the morphogram X and its quotation "X" in the normalization (diagonalization) X"X". The relationship between X and the quotation "X" is ruled by retro-grade recursive repetition modulo palindrome.

That is, not all retro-grade recursive repetitions "X" are producing a palindrome X"X".

Palindromicity is therefore a criterion to decide the correctness of quotational formulas in the use as normalizations and diagonalizations in morphic formal systems.

Normalization is related to *concatenation*, while diagonalization is related to *substitution*. Both kinds of iterability are ruled by complementary context conditions. The context conditions (rules) for normalization is palindromicity. The context rules for substitution are more complicated and had been sketched in earlier papers.

There might be other criteria for normalization and substitution for morphic systems.

Following the distinction of *even* and *odd* palindromes some further distinctions follow naturally: quotations with midpoint centre of different length, interpreted as neutral parts of the quotational process, and quotations without neutral parts.

Even, odd and symmetric, asymmetric, inverse and iterative/accretive modi of quotation.

Main modi of quotation
Quotation as *replication*: [1,2,2,3,1,2,2,3]: [head, tail=head],
Quotation as *inversion*: [1,2,2,3,3,2,2,1]: [head, tail=rev(head)],
Quotation as *accretion*: [1,2,2,3,4,2,2,5]: [head, tail=accr(head)].

Relationships between the modi of quotation
Repetition or replication is a kind of accretion without differentiation.
Repetition is kind of an inversion of an inversion.
Inversion is a special repetition of the reversed original word.
Repetition is a kind of inversion without reversion.
Accretion is repetition with differentiation.
Accreation is neither iteration nor inversion.

Order(Rep, Inv, Accr): Repetition $\subseteq$ Inversion $\subseteq$ Accretion.

Again, example Tcontexture 8:
All 14 possible X"X" palindromic constructions as quotations of X = [1,2,2,3]:

[1,2,2,3,3,1,1,2],
[1,2,2,3,2,3,3,1],
[1,2,2,3,1,2,2,3], [1,2,2,3,3,2,2,1],
[1,2,2,3,1,4,4,3], [1,2,2,3,3,4,4,1]
[1,2,2,3,4,1,1,2]
[1,2,2,3,4,2,2,1], [1,2,2,3,2,3,3,4]
[1,2,2,3,4,5,5,1]
[1,2,2,3,3,2,2,4]
[1,2,2,3,4,2,2,5]
[1,2,2,3,4,5,5,6]
[1,2,2,3,3,4,4,5].

Mirror-image preserving quotation as *inversion*: [1,2,2,3,3,2,2,1].

- ispalindrome [1,2,2,3,3,2,2,1];
val it = true : bool
- palindrome [1,2,2,3,3,2,2,1]; : symmetric palindrome
val it = true : bool

Direct quotation as *repetition*, X"X": [1,2,2,3,1,2,2,3].

- ispalindrome [1,2,2,3,1,2,2,3];
val it = true : bool
- palindrome [1,2,2,3,1,2,2,3];
val it = false : bool

Accretive quotation as asymmetric palindrome: [1,2,2,3,2,3,3,4].

- ispalindrome [1,2,2,3,2,3,3,4];
val it = true : bool

- [1,2,2,3] = tnf[2,3,3,4];
val it = true : bool

Modi for head=[1,2,2,3]
repetition: [1,2,2,3,1,2,2,3],
reversion: [1,2,2,3,3,2,2,1],
accretion: [1,2,2,3,2,3,3,1], [1,2,2,3,3,1,1,2],

[1,2,2,3,1,4,4,3],[1,2,2,3,3,4,4,1],[1,2,2,3,4,1,1,2],[1,2,2,3,4,2,2,1],[1,2,2,3,2,3,3,4], [1,2,2,3,3,2,2,4]
[1,2,2,3,4,5,5,1] ,[1,2,2,3,4,2,2,5],[1,2,2,3,3,4,4,5],
[1,2,2,3,4,5,5,6],

Read backwards, accretions are *reductions*: [1,2,2,3,4,5,5,6]: [4,5,5,6] ⇒ [1,2,2,3].

Examples for reduction from [4,1,2,3]: [1,2,3,4], [1,1,2,3] and [1,2,1,3]
[1,2,3,4,4,1,2,3],[1,1,2,3,4,1,2,3],[1,2,1,3,4,1,2,3].
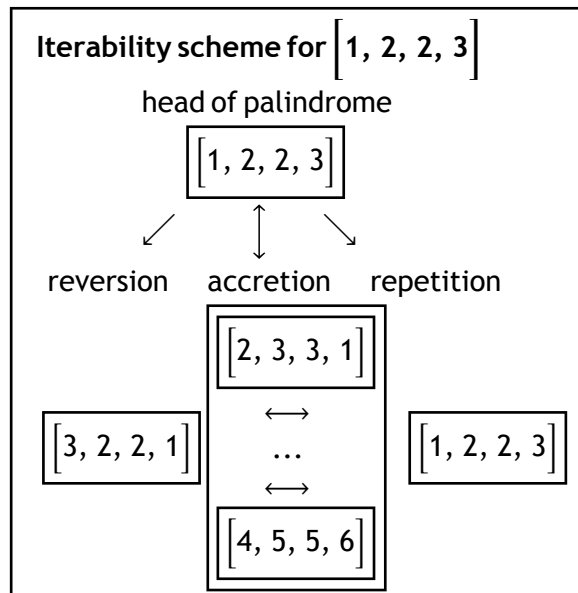
Without doubt there are some possibilities of a 'micro'-classification of the accretive mode of 'quotation'.

Accretion as negation
To build a simple connection to the classical expression of self-reference

and contradiction it seems to be appropriete to use a form of a permutative negation over the structure of the quote to replace negation.

Hence, a quotation of [1,2,2,3] combined with 'accretive' negation (permutation) becomes [1,4,4,3] with

neg[1,2,2,3] = [1,4,4,3], i.e. 1→1, 3→3 and 2→4.

Examples

Micro-classification of accretion

[1,2,2,3,1,4,4,3], [1,2,2,3,3,4,4,1]: tail1=inv(tail2)

[1,2,2,3,4,1,1,2],[1,2,2,3,4,2,2,1]: tail2=accr(tail1).

Balanced replication points: lst(head)=frst(tail)

[1,2,2,**3**,**3**,1,1,2],[1,2,2,**3**,**3**,4,4,1], [1,2,2,3,3,2,2,4],[1,2,2,3,3,4,4,5].

2.1.6. A palindromic field as a realization of the spatiality of palindromes

Each palindrome is morphogrammatically a *palindromic field* defined by its dynamic 'source' and its dimensions *reversion*, *repetition* and *accretion*. Single palindromes are members of their palindromic field.

With that, a more intertextual approach is emphazised. The metaphor of a *Möbius strip* could add to its understanding.
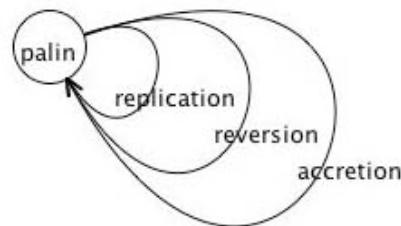
Example
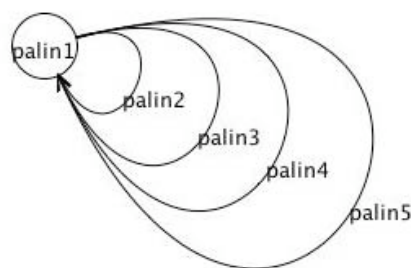
Part of the palindromic field of palin1 = [1,2,2,3]:

palin1 = [1,2,2,3], palin2 = [3,2,2,1], palin3 = [2,3,3,1], palin4 = [3,1,1,2], palin5 = [1,4,4,3].

Again, the cycles are in no way something like feed-back loops or similar iterations. Without introducing too much complications it shall
be remembered that the 'head' of the palindromes, $palin_i$, is not atomically defined in the equivalent way for all 'turns' but gets retro-recursively

involved by each of its turns at new with the added part palin$_i$.

This is well demonstrated by the ENstructure of the palindromes.

```
- ENstructure [1,2,2,3];       : EN(palin1)
val it = [[],[(1,2,N)],
           [(1,3,N),(2,3,E)],
           [(1,4,N),(2,4,N),(3,4,N)]]
 : (int * int * EN) list list
```

```
- ENstructure[1,2,2,3,3,2,2,1];
val it =
  [[],[(1,2,N)],[(1,3,N),(2,3,E)],[(1,4,N),(2,4,N),(3,4,N)],      : EN(palin1)
     [(1,5,N),(2,5,N),(3,5,N),(4,5,E)],
     [(1,6,N),(2,6,E),(3,6,E),(4,6,N),  (5,6,N)],
     [(1,7,N),(2,7,E),(3,7,E),(4,7,N),  (5,7,N),(6,7,E)],
     [(1,8,E),(2,8,N),(3,8,N),(4,8,N),  (5,8,N),(6,8,N),(7,8,N)]]

        EN(palin1 in [palin1, palin2])     EN(palin2)
```

```
- ENstructure[1,2,2,3,1,4,4,5];
val it =
  [[],[(1,2,N)],[(1,3,N),(2,3,E)],[(1,4,N),(2,4,N),(3,4,N)],      : EN(palin1)
     [(1,5,E),(2,5,N),(3,5,N),(4,5,N)],
     [(1,6,N),(2,6,N),(3,6,N),(4,6,N),  (5,6,N)],
     [(1,7,N),(2,7,N),(3,7,N),(4,7,N),  (5,7,N),(6,7,E)],
     [(1,8,N),(2,8,N),(3,8,N),(4,8,N),  (5,8,N),(6,8,N),(7,8,N)]]
        EN(palin1 in [palin1, palin5])     EN(palin5)
```

Thus, EN(palin1 in [palin1, palin2]) ≠ EN(palin1 in [palin1, palin5]).

$$
\begin{pmatrix}
\boxed{\begin{array}{l}
\text{palin1} = [1, 2, 2, 3] \\
[(1, 2, N)], \\
[(1, 3, N), (2, 3, E)], \\
[(1, 4, N), (2, 4, N), (3, 4, N)]
\end{array}} & \text{palin}(1, 5) = [1, 2, 2, 3, 1, 4, 4, 5] \\
\boxed{\begin{array}{l}
[(1, 5, E), (2, 5, N), (3, 5, N), (4, 5, N), \\
[(1, 6, N), (2, 6, N), (3, 6, N), (4, 6, N), \\
[(1, 7, N), (2, 7, N), (3, 7, N), (4, 7, N), \\
[(1, 8, N), (2, 8, N), (3, 8, N), (4, 8, N)] \\
\\
[\text{palin1, palin5}] = [1, 2, 2, 3, 1, 4, 4, 5]
\end{array}} & \boxed{\begin{array}{l}
\text{palin5} = [1, 4, 4, 5] \\
[(5, 6, N)], \\
[(5, 7, N), (6, 7, E)], \\
[(5, 8, N), (6, 8, N), (7, 8, N)]]
\end{array}}
\end{pmatrix}
$$
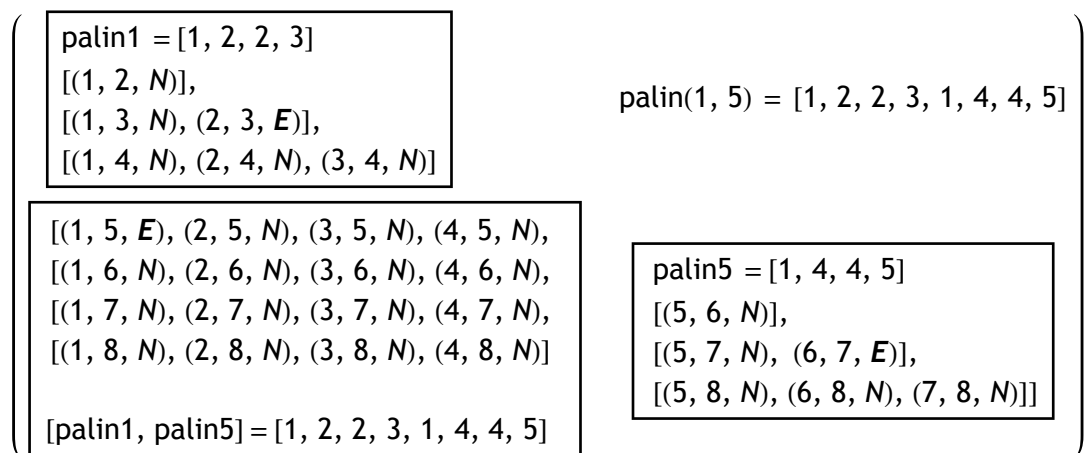
Hence, the head [1,2,2,3] gets different embedments into the whole palindrome, and therefore different characterizations, by each composed palindrome [palin1, palin$_i$]. The same holds obviously for the 'tail'.

Evolution/reduction of palindromic fields

Palindromic fields are what was metaphorically called the *'spatiality'* of (classical) palindromes.
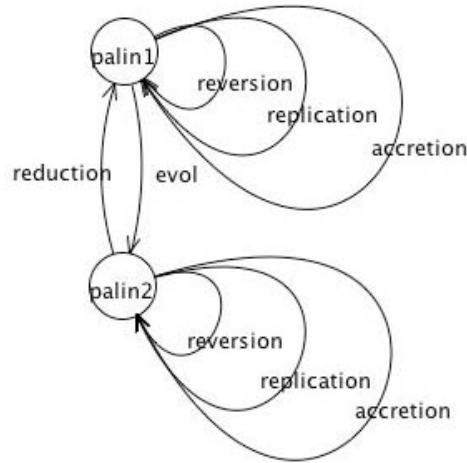
The whole field of palindromes of a contexture, *Contexture(n)*, is defined by the interaction (evolution/reduction) of all possible morphic palindromes generated by the different 'heads' (sources, morphograms) of a given contexture.

Example: Tcontexture 6

- Tcontexture 6;
- length it;
val it = 203 : int
List.filter ispalindrome "Contexture 6"
val it =
1. [[**1,1,1**,1,1,1], [1,1,1,2,2,2],
:2
2. [**1,1,2**,1,2,2],[1,1,2,2,1,1], [1,1,2,3,4,4],[1,1,2,2,3,3], [1,1,2,3,1,1],
: 5
3.
[**1,2,1**,2,1,2],[1,2,1,1,2,1],[1,2,1,3,2,3],[1,2,1,1,3,1],[1,2,1,3,4,3],[1,2,1,3,4,3],
: 6
4. [**1,2,2**,1,1,2],[1,2,2,2,2,1],[1,2,2,3,3,1],[1,2,2,2,2,3],[1,2,2,3,3,4],
: 5
5.
[**1,2,3**,1,2,3],[1,2,3,2,3,1],[1,2,3,3,1,2],[1,2,3,3,2,1],[1,2,3,4,1,2],[1,2,3,4,2,1],[
1,2,3,1,4,3], : 14

[1,2,3,3,4,1],[1,2,3,2,3,4],[1,2,3,3,2,4],[1,2,3,4,5,1],[1,2,3,4,2,5],[1,2,3,3,4,5],[1
,2,3,4,5,6]] : int list list
- length it;
val it = 31 : int

Tcontexture(6) opens up 5 fields of palindromes of length 6.

Number of fields for palindromes of length $= n : \sum_{k=1}^{m} Sn(k, m), \ m = n/2.$

There are 5 palindromic fields for 31
      palindromes of 203 morphograms from Tcontexture 6 :

$$\sum_{k=1}^{3} Sn(k, 3) = 1 + 3 + 1 = 5.$$

2.1.7. Special forms of quotation

For the case of *Tcontexture 8* there are interesting types of quotation patterns.

[**1,1,1,1**,1,1,1,1]: homogeneous replication
[1,1,1,1,2,2,2,2]: homogeneous accretion

[**1,2,3,4,**1,2,3,4]: heterogeneous replication

Heterogeneous inversions
[**1,2,3,4**,1,3,2,4],[1,2,3,4,2,1,4,3],[1,2,3,4,2,3,4,1],
[1,2,3,4,3,4,1,2],[1,2,3,4,3,4,2,1],[1,2,3,4,4,1,2,3],
[1,2,3,4,4,2,3,1],[1,2,3,4,4,3,1,2],
[1,2,3,4,4,3,2,1]: direct heterogeneous inversion,...

Heterogeneous accretions
[**1,2,3,4**,5,1,2,3],[1,2,3,4,5,2,3,1],[1,2,3,4,5,3,1,2],[1,2,3,4,5,3,2,1],...
[1,2,3,4,1,5,2,4],[1,2,3,4,2,5,4,1],[1,2,3,4,4,5,1,2],[1,2,3,4,4,5,2,1],...
[1,2,3,4,1,3,5,4],[1,2,3,4,3,4,5,1],[1,2,3,4,4,1,5,3],[1,2,3,4,4,3,5,1],...
[1,2,3,4,2,3,4,5],[1,2,3,4,3,4,2,5],[1,2,3,4,4,2,3,5],[1,2,3,4,4,3,2,5],...

[1,2,3,4,5,6,1,2],[1,2,3,4,5,6,2,1],...
[1,2,3,4,5,1,6,3],[1,2,3,4,5,3,6,1],...
[1,2,3,4,1,5,6,4],[1,2,3,4,4,5,6,1],**...**
[1,2,3,4,5,2,3,6],[1,2,3,4,5,3,2,6],...
[1,2,3,4,2,5,4,6],[1,2,3,4,4,5,2,6],...
[1,2,3,4,3,4,5,6],[1,2,3,4,4,3,5,6],

[1,2,3,4,5,6,7,1],[1,2,3,4,5,6,2,7],
[1,2,3,4,5,3,6,7],[1,2,3,4,4,5,6,7],
[1,2,3,4,5,6,7,8]: total heterogeneous accretion

The palindrome approach to quotation, self-reference and self-citation, i.e. self-referential cognition/action, is reflecting retro-grade recursive the intrinsic complexity of the "original" pattern of the quote and the modi of the complexity of the process of quotation.

Hence, the general, sentence-oriented formula for normalization <N"N">, becomes <compl$_i$(N) compl$_j$"N")>.

compl(N"N") = (compl$_i$(N), compl$_j$("N")), $1 \leq i, j \geq m$, m=length of palindrome.

The sentence-oriented formula <N"N"> has the morphic complexity-pattern [1,1]. It might be augmented in meta-language considerations to [1,2].

2.1.8. Morphic differentiation FSM for palindromes

Classical modeling with Push-Down automta for even and odd palindromes

Palindromes over $\Sigma = \{a, b\}$
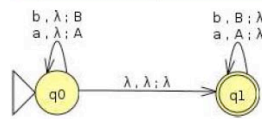
accepts $\{uu^R \in \Sigma^* \mid u \in \Sigma^*\}$ the even
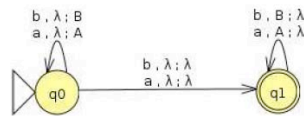
accepts $\{u\sigma u^R \in \Sigma^* \mid u \in \Sigma^*, \; \sigma \in \Sigma\}$
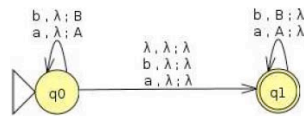
accepts the palindromes

ftp://ftp.cs.kun.nl/pub/CompMath.Found/12TA7.pdf

Examples

- ENstructure[1,2,2,3];
val it =
[[],
[(1,2,N)],
[(1,3,N),(2,3,E)],
[(1,4,N),(2,4,N),(3,4,N)]]
 : (int * int * EN) list list

ENstructure automaton table for MorphoFSM [1,2,2,3]

MorphoFSA [1, 2, 2, 3]	pos $_1$	pos $_2$	pos $_3$
pos $_1$	e3	v1	v4
pos $_2$	v2	–	v6
pos $_3$	v7	v5	–
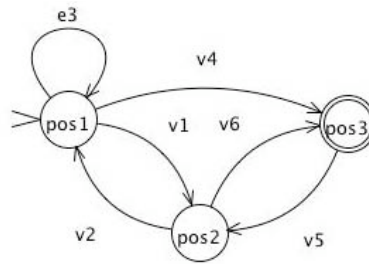
- ENstructure [1,2,2,3,3,1,1,2];
val it =
  [[],
  [(1,2,N)],
  [(1,3,N),(2,3,**E**)],
  [(1,4,N),(2,4,N),(3,4,N)],
  [(1,5,N),(2,5,N),(3,5,N),(4,5,**E**)],
  [(1,6,**E**),(2,6,N),(3,6,N),(4,6,N),(5,6,N)],
  [(1,7,**E**),(2,7,N),(3,7,N),(4,7,N),(5,7,N),(6,7,**E**)],
  [(1,8,N),(2,8,**E**),(3,8,**E**),(4,8,N),(5,8,N),(6,8,N),(7,8,N)]]
  : (int * int * EN) list list

ENstructure automaton table for MorphoFSM [1,2,2,3,3,1,1,2]

MorphoFSA [1, 2, 2, 3, 3, 1, 1, 2]	pos $_1$	pos $_2$	pos $_3$
pos $_1$	e3	v1	v4
pos $_2$	v2	e15	v6
pos $_3$	v7	v5	–

- ENstructure[1,2,2,3,4,5,5,6];
val it =
  [[],
  [(1,2,N)],
  [(1,3,N),(2,3,**E**)],
  [(1,4,N),(2,4,N),(3,4,N)],
  [(1,5,N),(2,5,N),(3,5,N),(4,5,N)],
  [(1,6,N),(2,6,N),(3,6,N),(4,6,N),(5,6,N)],
  [(1,7,N),(2,7,N),(3,7,N),(4,7,N),(5,7,N),(6,7,**E**)],
  [(1,8,N),(2,8,N),(3,8,N),(4,8,N),(5,8,N),(6,8,N),(7,8,N)]]
  : (int * int * EN) list list

ENstructure automaton table for MorphoFSM [1,2,2,3,4,5,5,6]

MorphoFSA [1, 2, 2, 3, 4, 5, 5, 6]	pos $_1$	pos $_2$	pos $_3$	pos $_4$	pos $_5$	pos $_6$
pos $_1$	e3	v1	v4	v7	□	□
pos $_2$	v2	v6	v6	□	□	□
pos $_3$	□	v5	□	□	□	□
pos $_4$	□	□	□	□	□	□
pos $_5$	□	□	□	□	□	□
pos $_6$	□	□	□	□	□	□

2.1.9. Linguistic and literary examples of modi of quotation

Creative mirror literature (Oskar Panizza) must be full of unconventional quotations on all levels of the linguistic tectonics.

See *"On Self-referential Sentences"* in: Douglas R. Hofstadter, Metamagical Themas.", 1981

Kenny's Overview of Hofstadter's Explanation of Gödel's Theorem
http://www4.ncsu.edu/unity/lockers/users/f/felder/public/kenny/papers/godel.html

Henry W. Pickford, Heimrad Bäcker's "System nachschrift" and the Philosophy of Quotation, 2008
"Thus Bäcker writes of quotation as "deckungsgleiche Wiedergabe": "ich erfasse die Möglichkeit, durch die isolierung der reihe und ihre deckungsgleiche Wiedergabe eine konkretion zu bilden: konkret im Hinblick auf die isolierte form, konkret aber auch im Blick auf den Vorgang jener nacht und sein Tremedum" ("Mauthausen" 128).
http://gsll.colorado.edu/sites/default/files/images/stories/facultymedia/backer.pdf

Steven Rendall, Translation, Quotation, Iterability
"Iterability, the possibility of repetition, is that which, within any utterance, exceeds the utterance as such, escapes any given spatial, temporal, or intentional context, and points toward the possibility of saying (writing, reading) "the same thing" in an infinite variety of other contexts.
For Derrida, iterability is a constitutive feature of language; all language is always in principle iterable, and a word that could not be repeated would simply not be a word.

"discovered in quotation the power, not to preserve, but to purify, to rip out of context, to destroy; the only power in which there remains any hope that something will outlast this era — precisely because it has

been wrenched out of it."
http://www.erudit.org/revue/TTR/1997/v10/n2/037303ar.pdf

Analytic theories of quotation
Jan Schreiber, A Bibliography on Quotation and the Use-Mention Distinction (2008)
http://www.uni-due.de/~gph120/Bib/Bibliography.html

2.2. Computability with morphic palindromes

2.2.1. Turing Machines

"The key property of Turing machines, and all other equivalent models of computation, is *universality*: there is a single Turing machine U that is capable of simulating any other Turing machine — even those with vastly more states than U . In other words, one can think of U as a *"Turing machine interpreter,"* written in the language of Turing machines.

"This capability for self-reference (the language of Turing machines is expressive enough to write an interpreter for itself) is the source of the surprising versatility of Turing machines and other models of computation."
http://www.cs.cornell.edu/courses/cs4820/2012sp/handouts/turingm.pdf

The analogy to diagonalization/normalization continues with the famous self-application for universal Turing Machines UTM:
   < UTM as a *"Turing machine interpreter,"* written in the language of Turing machines >.

Scheme
Turing machine description,
Turing machine interpreter,
Turing machine interpreter of "Turing machine description of".

The mechanism of the analogy
"The proof proceeded in two steps.
Step 1 is to suppose (for *reductio ad absurdum*) that the sequence S is a complete enumeration of all the members of M .
Step 2 is to employ the *diagonal method* to generate a sequence that belongs to M but does not appear on the list S.

"In other words, they share the same general scheme by which a contradiction results from an enlargement of a set by *constructed-from-the-outside* entities." (Tworak)

Strategy

Step 1

Accept the dominance of logic. (Holds for direct and indirect proofs.)

Step2

Apply creatively the *ad hoc* trick(of diagonalization or normalization).

Step 3

Be happy to have constructed on the base of the two presumptions a logical *contradiction* that accepts the *"reduction ad absurdum"* principle.

Step4

Take your trophy home until it is too late.

Subversion: to-do list

If you are not happy with this traditionally proved strategy,

firstly, ask for the legitimacy of the presupposition of the dominant logic (even tradition offers different approaches).

Then, study with praxeological strictness what exactly happens step-by-step by the *application* of the "creative trick".

Don't be perplexed, there is no reason to freak out, if your description of the game leads you beyond the narrowness of your learned rationality.

This opens up a new job to do: elaborate the game of *dissemination* and *mediation* of the tabular constellation of the description beyond the framework of classical logic, formal systems and semiotics.

If you feel forced to move back home, it will be just a simple exercise to reduce and resume the structural complexity under the established umbrella. And you will be save again.

Comment

Don't forget that from a *praxeological* point of view, a number is a number if and only if it is defined by its rules of construction, and not by its 'objectional' product (result) that is denying any history of its construction. Hence, the saying that all numbers are listed in the table is what it is, the list of all numbers defined by the given procedure. And nothing else, not more and not less.

A number defined by *diagonalization* has a totally different definition than the numbers of the list and there is therefore no surprise that this totally differently defined number is not a member of the list of the numbers defined by the first definition of numbers. The numbers of the list of the listed numbers are not defined by the "trick" (or reflection) of diagonalization over the listed numbers defined by the first definition of numbers.

Hence, the *paradox* that there is a number not entailed in the list of all listed numbers does the job only if its praxeological properties are denied

and only the 'object' number is accepted as a number, i.e. as a numeral - and nothing else. With that we might speak about "all numbers" (independently of their definition).

If the process of the definition of the number is conceived and accepted as crucial, the whole surprise and its paradoxical stumbling falls apart.

There are simply just two kinds of numbers defined in Cantor's game: the *listed* numbers and the *diagonal* number(s) of the listed numbers. After that, a logical yes/no-comparison follows.

The concept of a "diagonal number" is therefore relative. There is not just one diagonal number but infinite many diagonal numbers depending on the variable definition of the 'diagonal' of the listed numbers.

Other constructions are freely possible too.

Again, who is afraid of this *praxeological* approach of two discontexturally different natural number series (NNS) and also doesn't want to take the burden of a new step of formalization, always has the possibility to freeze the process and to turn back to the text-book knowledge.

Short double check: the construction of Cantor's Diagonal Method following Zbigniew Tworak

Theorem 1. *No denumerable sequence of elements of the set M contains all elements of this set*.

Proof. Let S = <E_1 , E_2 , . . . , E_j , . . . > be any denumerable infinite sequence of members of the set *M* . That is *S* is sequence of sequences. The table below is an illustration of this:

		1	2	3	...
E_1		a_{11}	a_{12}	a_{13}	...
E_2		a_{21}	a_{22}	a_{23}	...
E_3		a_{31}	a_{32}	a_{33}	...
⋮		⋮	⋮	⋮	

"The sequence E_0 has following properties: (a) it belongs to *M* (because its each element is either *m* or *w*), (b) for a natural number k, $E_0 \neq E_k$ (E_0 differs from E_1 in the first digit, from E_2 in the second digit, and so on), i.e. E_0 does not appear on the list *S*. Thus the supposition that *S* is a complete enumeration of all the members of *M* is false. In consequence, the set *M* is *non-denumerable*."

"The *proof* proceeded in two steps.
Step 1 is to suppose (for *reductio ad absurdum*) that the sequence S is a complete enumeration of all the members of *M* .

Step 2 is to employ the *diagonal method* to generate a sequence that belongs to *M* but does not appear on the list *S*.

"To this end we take into consideration the diagonal sequence $<a_{11}, a_{22}, \ldots, a_{kk}, \ldots>$ and generate a new sequence $E_0 = <b_1, b_2, \ldots, b_k, \ldots>$, the anti-diagonal sequence, as follows: if $a_{kk} = m$ then $b_k = w$, and if $a_{kk} = w$ then $b_k = m$.

"The two tables below are picturing this procedure:

The table		1	2	3	...	convers		1	2	3	...
E_1		m	a_{12}	a_{13}	...		E_1	w	a_{12}	a_{13}	...
E_2		a_{21}	w	a_{23}	...		E_2	a_{21}	m	a_{23}	...
E_3		a_{31}	a_{32}	m	...		E_3	a_{31}	a_{32}	w	...
$\vdots$		$\vdots$	$\vdots$	$\vdots$			$\vdots$	$\vdots$	$\vdots$	$\vdots$	

"(The tables above show why this method is called the *diagonal method* : we have constructed a new sequence by changing each element on the diagonal.)"

(Tworak, Logic and Logical Philosophy, Volume 15 (2006), 39–67)

http://www.logika.umk.pl/llp/151/151-3.pdf

Further elaborations

Noson S.Yanofsky, A Universal Approach to Self-Referential Paradoxes, Incompleteness and FixedPoints, 2003

http://xxx.lanl.gov/pdf/math/0305282v1.pdf

2.2.2. Diagonalization/Normalization

"By the *norm* of an *expression* X, we shall mean the expression X(X) — e.g. the norm of P~ is P~(P~). By a *sentence*, we mean any expression of one of the following four forms (X is any expression):

(1) P(X)

(2) PN(X)

(3) ~ P(X)

(4) ~ PN(X)

Informally, P stands for "printable"; N stands for "the norm of" and ~ stands for "not". And so we define P(X) to be true if (and only if) X is printable. We define PN(X) to be true if the norm of X is printable. We call ~ P(X) true iff (if and only if) X is not printable, and ~ PN(X) is defined to be true iff the norm of X is not printable."

Print morphogram [M]: print([M])

print norm of [M]: print(norm([MG]))

norm of [MG] : [MG]"[MG]" : quotation

norm of [MG]: repetition of [MG]: R*[MG] = [MG]"[MG]".

Print([M])
print(norm([M]))
non(print([M]))
non(print(norm([M])))

> *"Also if PR*X is printable, it is true, which means that XX must be printable. Now, suppose that X is printable; does it necessarily follow that P*X is printable? No; if X is printable then P*X is certainly true, but we are not given that the machine is capable of printing all true sentences, but only that the machine never prints any false ones."*

Corresponcences

"(1) P*X is true if X is printable." This corresponds to: [MG]*[MG] is palindromic if [MG*MG] is realizable.

"(2) PR*X is true if XX (the repeat of X) is printable." This sentence corresponds to the morphogrammatic situation of

[MG]*[MG] is palindromic if [MG*MG] is realizable.

PR*X is true $\cong$ [MG]*[MG] is palindromic
if XX is printable $\cong$ [MG*MG] is realizable.

"if X is printable, (PX), then P*X is certainly true" $\cong$ "if [MG] is realizable then [MG*MG] is palindromic.

A morphogrammatic system (machine, grammer) is producing morphograms.

Palindromes are special morphograms of the morphogrammatic system (scripture).

2.2.3. Modeling with "Gödel's Games"

To interpret is to unquote. Hence, interpretation and quotation, I and Q, are symmetric operations on sentences and programs.

Because morphograms are not sequences there is no need for negation to mark the difference between the results of the quote/interpret mechanism. The morphogrammatic mechanism of quotation and interpretation happens in the framework, i.e. under the conditions of palindromicity.

Strategy   **Model**

$$Q^1\left(G^1\right) \longrightarrow G^2 \quad : \quad Q^1\left(\left[1, 2, 2, 3\right]\right) \longrightarrow G^2 = \left[1, 2, 2, 3, \mathbf{3, 2, 2, 1}\right]$$

$$G^1 \longleftarrow I^2\left(G^2\right) \quad : \quad G^1 = \left[\mathbf{3, 2, 2, 1}\right] \longleftarrow I^2\left(\left[1, 2, 2, 3, 3, 2, 2, 1\right]\right)$$

$$G^2 \longleftarrow I^3\left(G^3\right) \quad : \quad G^2 = \left[\mathbf{1, 4, 4, 3}\right]\right) \longleftarrow I$$

$$^3\left(G^3\right) = \left[1, 2, 2, 3, 1, 4, 4, 3\right]$$

$$Q^2\left(G^2\right) \longrightarrow G^3 \quad :$$

$$Q^2\left(G^2\right) = \left[1, 2, 2, 3\right] \longrightarrow G^3 = \left[1, 2, 2, 3, \mathbf{1, 4, 4, 3}\right]$$

$$G^1 \longleftrightarrow G^2 \quad : \quad G^1 = \left[3, 2, 2, 1\right] \longleftrightarrow G^2 = \left[1, 4, 4, 3\right]$$

I : Interpretation, Q : Quotation, G : Sentence (morphogram)

Strategy   **and**   **Explanation**

$$Q^1\left(G^1 = \text{stated morphogram}\right) \quad \longrightarrow G^2 =$$
$$\text{morphic palindrome of } Q^1\left(G^1\right)$$

$$G^1 = \text{quoted morphogram}\left(\text{of } G^1\right) \longleftarrow I^2\left(G^2 = \text{morphic palindrome}\right)$$

$$G^2 = \text{quoted morphogram}\left(\text{of } G^2\right) \longleftarrow I^3\left(G^3 = \text{morphic palindrome}\right)$$

$$Q^2\left(G^2 = \text{stated morphogram}\right) \quad \longrightarrow G^3 = \text{morphic palindrome of } Q^2$$

$$G^1 = \text{quoted morphogram}\left(\text{of } G^1\right) \longleftrightarrow G^2 =$$
$$\text{quoted morphogram}\left(\text{of } G^2\right)$$

I : Interpretation, Q : Quotation, G : Sentence (morphogram)

Description of the procedure

Start with a morphogram, G^1.

       2,   1,

    1,   2 =   1   1"

   2   2   2   2).

                  2,

Build a morphic palindrome, G^2, based on the start morphogram, G^1, by quotation, Q^1, of that morphogram, $G^2 = G^{1"}G^{1"}$.

Interprete, I^2, this palindromic morphogram, G^2, $I^2(G^2)$.

Note the quoted part of the interpreted palindromic morphogram, G^2, by "$G^{1"}$.

Start with a morphogram, G^2, equivalent to the morphogram G^1.

Build a morphic palindrome, G^3, based on the start morphogram, G^2 by quotation, Q^2, $Q^2(G^2)$.

Interprete, I^3, this palindromic morphogram, G^2, $I^3(G^2)$.

Note the quoted part of the interpreted palindromic morphogram, G^3, by "$G^{2"}$.

Compare both results of the quotation/interpretation process: G^1 and G^2.

Note the difference (differentiation, distinction) between G^1 and G^2.

Result

There is no logical contradiction between "$G^{1"}$ and "$G^{2"}$ but a *difference* that is separating both morphic palindromes.

As morphograms, both quotations are morphogrammatically equivalent, $G^1 =_{MG} G^2$, but semiotically (symbolically) they are different. Nevertheles, the compound morphograms G^1 and G^3 are morphogrammatically different.

Strategy	Model

$Q^1_{pal-1}(G^1) \longrightarrow G^1 \text{"} G^1_1 \text{"} \quad :$

$\qquad Q^1_1([1, 2, 2, 3]) \longrightarrow G^1 \text{"} G^1 \text{"} = [1, 2, 2, 3, \mathbf{3, 2, 2, 1}] \in \text{Pal}$

$\quad | \qquad\qquad | \qquad\qquad\qquad\qquad | \qquad\qquad\qquad\qquad\qquad |$

$\quad \text{"} G^1_1 \text{"} \longleftarrow I^2(G^1 \text{"} G^1_1 \text{"}) \quad :$

$\qquad\qquad \text{"} G^1 \text{"} = [\mathbf{3, 2, 2, 1}] \longleftarrow I^2([1, 2, 2, 3, 3, 2, 2, 1])$

$\text{"} G^1_2 \text{"} \longleftarrow I^3(G^1 \text{"} G^1_2 \text{"}) \quad : \text{"} G^2 \text{"} = [\mathbf{1, 4, 4, 3}]) \longleftarrow I$

$\qquad ^3(G^3) = [1, 2, 2, 3, 1, 4, 4, 3]$

$| \qquad\qquad\qquad | \qquad\qquad\qquad\qquad | \qquad\qquad\qquad\qquad\qquad\qquad |$

$Q^1_{pal-2}(G^1) \longrightarrow (G^1 \text{"} G^1_2 \text{"}) \quad :$

$\quad Q^1_2(G^1) = [1, 2, 2, 3] \longrightarrow G^3 = [1, 2, 2, 3, \mathbf{1, 4, 4, 3}] \in \text{Pal}$

$\text{"} G^1_1 \text{"} \quad \longleftrightarrow \quad \text{"} G^1_2 \text{"} \qquad\qquad : \quad \text{"} G^1_1 \text{"} = [3, 2, 2, 1] \longleftrightarrow \text{"} G^1_2 \text{"} = [1, 4, 4, 3]$

I : Interpretation, Q : Quotation, G : Sentence (morphogram)

Different strategies

Replication in the mode of identity : $\quad Q^1_{pal-1}(G^1) \longrightarrow G^1 \text{"} G^1_1 \text{"} : \text{"} G^1_1 \text{"} =$

$\qquad _{sem} \text{repl}(G^1),$

Reversion in the mode of identity : $\quad Q^1_{pal-1}(G^1) \longrightarrow G^1 \text{"} G^1_1 \text{"} : \text{"} G^1_1 \text{"} =$

$\qquad _{sem} \text{rev}(G^1),$

Accretion in the mode of difference : $\quad Q^1_{pal-2}(G^1) \longrightarrow G^1 \text{"} G^1_1 \text{"} : \text{"} G^1_1$

$\qquad \text{"} =_{MG} \text{accr}(G^1).$

Theorem

Classical quotation, $X \text{"} X \text{"}$, is trivially palindromic for reversion.

3. Formal modeling of self-reference

3.1. Modeling with the Lambda Calculus approach

3.1.1. General model

Proof of the Fixed Point Theorem by Barendregt:

Barendregt' proof

$\forall F \exists X\ FX = X.$

$Y \equiv (\lambda f. (\lambda x.f(xx))(\lambda x.f(xx)))$ such that

---

Barendregt' proof

$\forall F \exists X\, FX = X.$
$Y \equiv (\lambda f.\,(\lambda x.f\,(xx))\,(\lambda x.f\,(xx)))$ such that
$\forall F\, F\,(YF) = YF.$

Define $W \equiv \lambda x.\, F\,(xx)$ and $X \equiv WW.$
Then
$X \equiv WW \equiv \lambda x.\, F\,(xx)\, W \equiv F\,(WW) \equiv FX$

Thus, the full scheme of the Fixedpoint theorem is now marked by its numbers of occurrence of the term "W" in the application. Following Wittgenstein's advise that the meaning of a term is defined by its *use*, at least 6 different use of the term "W" are to distinguish in Barendregt's proof.

Fixed – point scheme

Define $W^1 \equiv \lambda x.\, F\,(xx)$ and $X \equiv W^2 W^3.$
Then
$X \equiv W^2 W^3 \equiv \lambda x.\, F\,(xx)\, W^4 \equiv F\left(W^5 W^6\right) \equiv FX$

With Barendregt's formula for the construction of the *Fixed Point Theorem*, the term "W" is used 6 times and therefore we have to check all its possible ways of use. The comfortable excuse to use the distinction of a syntactic and a semantic or of an object- and meta-language use of the terms, say 'use' and 'mention', to avoid further analysis, isn't of leading importance in this case.

There are at least 4 possible types of modeling:
1. identitive *equality* (semiotics),
2. kenomic *equivalence* (kenogrammatics),
3. morphic *similarity* (morphogrammatics),
4. metamorphic *bisimilarity* (metamorphosis).

http://memristors.memristics.com/Church-Rosser%20Morphogrammatics/Church-Rosser%20in%20Morphogrammatics.html
http://www.thinkartlab.com/pkl/lola/poly-Lambda_Calculus.pdf

3.1.2. Semiotic model: identitive equality

$W1 \equiv \lambda x.F(xx)$, $X \equiv W2W3$, $\lambda x.F(xx)W4$, $F(W5W6)$.

Example0: *Identity* : type[1,1,1,1,1,1]

Table : $u \underset{\text{ident}}{\Longrightarrow} v$

$u \underset{\text{ident}}{\Longrightarrow} v$	w_{12}	w_{13}	w_{23}	X1X2
MG	+	+	+	+
SEM	+	+	+	+
⌢	+	+	+	+
length	+	+	+	+

The most obvious use of "W" is the use in the mode of identity:

Thus, for all i, j, Wi, Wj: $Wi \equiv Wj$, with $i,j \in \{1, \ldots, 6\}$ and equal length: $length(Wi) = length(Wj)$, i.e. $length(+)$.

The kenogrammatic equivalence, MG (+), is automatically accepted by the semiotic equality, SEM(+).

Hence,

$\forall i, j$: Wi, Wj: $Wi \equiv Wj$, with $i,j \in \{1, \ldots, 6\}$

=>

$X1 \equiv W1W1 \equiv \lambda x.F(xx)W1 \equiv F(W1W1) \equiv FX1$.

That is the classical case of identity:

$X \equiv WW \equiv \lambda x.F(xx)W \equiv F(WW) \equiv FX$.

=> $X \equiv FX$.

Logical interpretation

$X \equiv WW \equiv \lambda x.non(xx)W \equiv non(WW) \equiv nonX$.

=> $X \equiv nonX$.

$\{X, nonX\} \in$ Contradiction.

Morphogrammatic palindromic interpretation

$W1 \equiv [1,2,2,3]$,

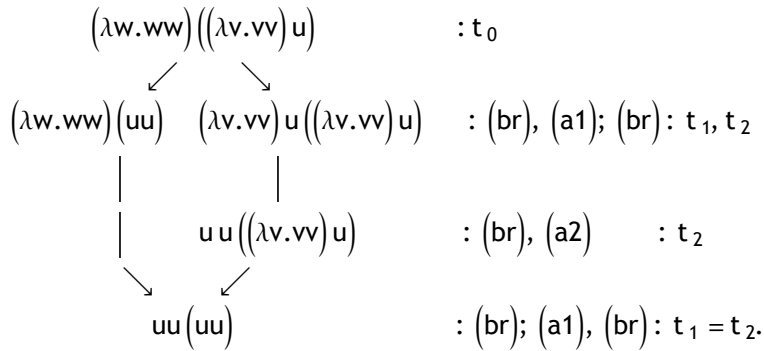$W1W1 = [1,2,2,3,1,2,2,3] = F(W1W1) = F[1,2,2,3,1,2,2,3]$: replication.

=> $X \in palin \equiv F(X) \in palin$.

$F = perm_{(1,3)}$: $perm_{(1,3)}(W1W1) = [3,2,2,1,3,2,2,1]$

=> $X \equiv_{keno} perm(X)$

Church – Rosser for equality

$$(br): \text{subst}: (\lambda v\, t)\, s \to t\left[v/s\right]_{ID}$$

$$(\lambda w.ww)\left((\lambda v.vv)\, u\right) \qquad\qquad : t_0$$

$$(\lambda w.ww)(uu) \quad (\lambda v.vv)\, u\left((\lambda v.vv)\, u\right) \quad : (br),\ (a1);\ (br): t_1, t_2$$

$$u\, u\left((\lambda v.vv)\, u\right) \qquad : (br),\ (a2) \qquad : t_2$$

$$uu(uu) \qquad\qquad : (br);\ (a1),\ (br): t_1 = t_2.$$

3.1.3. Palindromic model for kenomic equivalence

Scheme: $W1 \equiv \lambda x.F(xx)$, $X \equiv W2W3$, $\lambda x.F(xx)W4$, $F(W5W6)$.

Example1: type[1,1,2,4,1,3]: kenomic *equivalence*
W1,2,3,4,5,6
W1,1,2,4,1,3.

Table: $u \underset{equi}{\Longrightarrow} v$

$u \underset{equi}{\Longrightarrow} v$	w_{12}	w_{13}	w_{23}	X1X3
MG	+	+	+	−
SEM	−	−	−	−
⌒	+	+	+	+
length	+	+	+	+

$\lambda x.F(xx)$: palindromic replication (retro-grade recursive), i.e.
W2 is retro-recursively depending on W1, (and is thus not just a identive replication of W1) and on the palindromicity of the composition of W1 and W2 to W1W2.

$W1 \equiv \lambda x.F(xx)$, $X1 \equiv W1W2$, $\lambda x.F(xx)W1$, $F(W1W3) = F(X3)$.

$W1 \equiv [1,2,2,3]$,
$W2 \equiv [3,2,2,1]$, reversion of W1,
$W3 \equiv [1,4,4,3]$, accretion of W1,
$X1 \equiv [1,2,2,3,3,2,2,1] = W1W2$,
$X3 \equiv [1,2,2,3,1,4,4,3] = W1W3$.

$W1 \equiv \lambda x.F(xx)$, $X1 \equiv W1W2$, $\lambda x.F(xx)W1$, $F(W1W3)$:
$W1 \equiv \lambda x.F(xx)$

keno   keno

keno

$X1 \equiv W1W2 \cong_{keno} \lambda F.(xx)W1 \equiv F(W1W3) \cong_{keno} FX3$

$\Rightarrow X1 \cong_{keno} FX3$,

$\Rightarrow$ comparison: comp(X1, FX3) = separation, palindromic.

Comment

W2 and W3 are morphic 'replications' of W1. Hence, they are representing morphogrammatically different 'quotations' of W1.

W1, W2 and W3 are semiotically different, SEM (-), but morphogrammatically equivalent:

w_{12}: MG (+): $W1 =_{MG} W2$,

w_{13}: MG (+): $W1 =_{MG} W3$,

w_{23}: MG (+): $W2 =_{MG} W3$.

Semiotic identity, SEM(+), is requiered for identitive equality and identitive symmetric palindromes.

Also W2 and W3 are quotations of W1, the compositions X1 = W1W2 and X2 = W1W3 are morphogrammatically

not equivalent: X1X2: MG(-): $X1 \neq_{MG} X2$.

But both composed morphograms, X1 and X2, are palindromic: X1, X2 $\in$ Palin.

Hence, the comparison of X as X1 and X as X3 with FX3, comp(X1, FX3), is not a negation but marks a *separation*.

Because there are more different quotations of W1, the mapping between W1 and its quotations is not one-to-one but one-to-many.

The kenomic equivalence is founded in the structural, i.e. kenomic equivalence of the parts W2 and W3. (This is not anymore the case for the mode of kenomic *similarity*.)

Nevertheless, all the considerations in the mode of kenomic equivalence are based on the *same-length presupposition* of the morphograms: length-(X1, X3) = (+). (This is not anymore the case for the mode of the metamorphosis of kenomic *bisimilarity* where the length of X1 and X2 might differ.)

Another simple and balanced example is given by:

Example1': [1,1,1; 4,4,4]

W1,2,3,4,5,6

W1,1,1,4,4,4.

$W^1 \equiv W^2 \equiv W^3, W^4 \equiv W^5 \equiv W^6$

$W^1 \equiv \lambda x. F(xx), X \equiv W^1W^1, \lambda x. F(xx)W^4, F(W^4W^4)$:
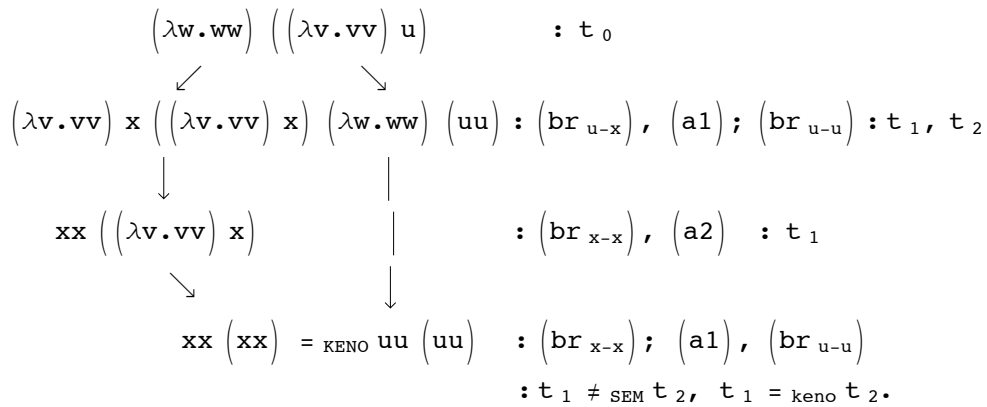
$W^1 \equiv \lambda x. F(xx)$

$X^1 \equiv W^1W^1 \cong_{keno} \lambda x. F(xx)W^4 \equiv F(W^4W^4) \cong_{keno} FX^4$

$\Rightarrow X^1 \cong_{keno} FX^4.$

Church – Rosser for equivalence

$$\text{Terms} = \left\{ \text{u, v, w, x, y, z} \right\}$$

$$\left(\text{br}_{keno} \right) : \left(\lambda \, \text{v} \, \text{t} \right) \, \text{s} \to \text{t} \left[\text{v} \big/ \text{s} \right]_{keno}$$

$$\left(\lambda \text{w.ww} \right) \left(\left(\lambda \text{v.vv} \right) \text{u} \right) \qquad : \text{t}_0$$

$$\left(\lambda \text{v.vv} \right) \text{x} \left(\left(\lambda \text{v.vv} \right) \text{x} \right) \left(\lambda \text{w.ww} \right) \left(\text{uu} \right) : \left(\text{br}_{u-x} \right), \left(\text{a1} \right); \left(\text{br}_{u-u} \right) : \text{t}_1, \, \text{t}_2$$

$$\text{xx} \left(\left(\lambda \text{v.vv} \right) \text{x} \right) \qquad\qquad : \left(\text{br}_{x-x} \right), \left(\text{a2} \right) \; : \text{t}_1$$

$$\text{xx} \left(\text{xx} \right) =_{KENO} \text{uu} \left(\text{uu} \right) \; : \left(\text{br}_{x-x} \right); \left(\text{a1} \right), \left(\text{br}_{u-u} \right)$$

$$: \text{t}_1 \neq_{SEM} \text{t}_2, \; \text{t}_1 =_{keno} \text{t}_2.$$

http://memristors.memristics.com/semi-Thue/Notes%20on%20semi-Thue%20systems.pdf
http://memristors.memristics.com/Church-Rosser%20Morphogrammatic-s/Church-Rosser%20in%20Morphogrammatics.html

3.1.4. Palindromic model for kenomic similarity

Example2: type[1,1,2,4,4,5]: palindromic *similarity*
W1,2,3,4,5,6
W1,1,2,4,4,5.
Table: $u \underset{simil}{\Longrightarrow} v$

$u \underset{simil}{\Longrightarrow} v$	w_{12}	w_{14}	w_{24}	w_{45}	w_{15}	X1X3
MG	+	−	+	+	−	−
SEM	−	−	−	−	−	−
⌢	+	+	+	+	+	+
length	+	+	+	+	+	+

λx.F(xx): palindromic replication (retro-grade recursive)

W1 $\equiv$ λx.F(xx), X1 $\equiv$ W1W2, λx.F(xx)W4, F(W4W5) = F(X3).

W1 $\equiv$ [1,2,2,3],
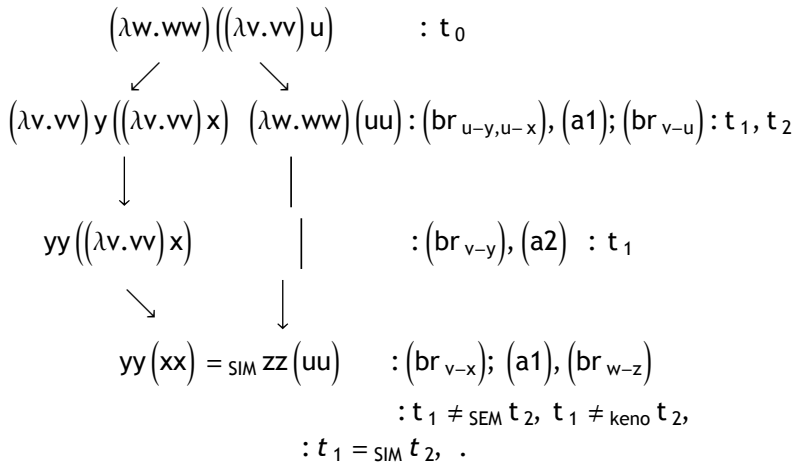W2 $\equiv$ [3,2,2,1],
W4 $\equiv$ [1,2,3,1],
W5 $\equiv$ [2,3,1,2],
X1 $\equiv$ [1,2,2,3,3,2,2,1] = W1W2, (inversion),

$X3 \equiv [1,2,3,1,2,3,1,2] = W4W5$, (accretion).

$W1 \equiv \lambda x.F(xx)$, $X1 \equiv W1W2$, $\lambda x.F(xx)W4$, $F(W4W5)$:

$W1 \equiv \lambda x.F(xx)$

$X1 \equiv W1W2 \cong_{keno} \lambda F.(xx)W1 \equiv F(W4W5) \cong_{keno} FX3$

$\Rightarrow X1 \cong_{keno} FX3$,

$\Rightarrow$ comparison: comp(X1, FX3) = separation, palindromic.

Church – Rosser for similarity

$\text{Terms} = \{u, v, w, x, y, z\}$

$\text{subst} : (\lambda v\, t)\, s \rightarrow t\left[v/s\right]_{SIM}$

$$(\lambda w.ww)\left((\lambda v.vv)u\right) \qquad : t_0$$

$$(\lambda v.vv)y\left((\lambda v.vv)x\right) \quad (\lambda w.ww)(uu) : (br_{u-y, u-x}), (a1); (br_{v-u}) : t_1, t_2$$

$$yy\left((\lambda v.vv)x\right) \qquad\qquad\qquad : (br_{v-y}), (a2) \; : t_1$$

$$yy(xx) =_{SIM} zz(uu) \quad : (br_{v-x}); (a1), (br_{w-z})$$

$$: t_1 \neq_{SEM} t_2, \; t_1 \neq_{keno} t_2,$$

$$: t_1 =_{SIM} t_2, \; .$$

3.1.5. Palindromic model for kenomic bisimilarity

Example3: type[1,1,1;4;5;6;7]: palindromic *bisimilarity*

Morphic bisimilarity abandons the security of the process of substitution/normalization established by the presumption in charge for the identity, the equivalence and the similarity option, of the same "length" of the substituted terms.

Hence, the situation "$W^1 \equiv \lambda x.\, F(xx)$" might be replaced by the *bisimilar* term "$\lambda x.\, F(xxx)$" delivering not WW but WWW with the abstraction $W^1 W^1 \cong_{bis} W^5 W^6 W^7$.

$u \underset{bis}{\Longrightarrow} v$	W_{12}	W_{14}	W_{23}	W_{45}	W_{15}	X1X3
MG	+	−	+	+	−	−
SEM	−	−	−	−	−	−
⌢	+	+	+	+	+	−
length	−	−	−	+	+	−

$W^1 \equiv W^2 \equiv W^3, W^4, W^5, W^6, W^7$

$\quad 1 \qquad \lambda$

$W^1 \equiv \lambda x.\ F(xx)$

$X^1 \equiv W^1 W^1 \cong_{bis} \lambda x.\ F(xxx)W^4 \cong_{sim} F(W^5 W^6 W^7) \cong_{sim} FX^{5.6}\quad .^7$

$\Rightarrow X^1 \cong_{bis} FX^{5.6}\quad .^7.$

Church – Rosser for bisimilarity

$\text{Terms} = \{u,\ v,\ w,\ x,\ y,\ z\}$

$$\left(br_{BIS}\right) : \left(\lambda v\, t\right)s \to t\left[v\big/s\right]_{BIS}$$

$$\left(\lambda w.ww\right)\left(\left(\lambda v.vv\right)u\right) \qquad\qquad :\ t_0$$

$$\left(\lambda v.vvv\right)y\left(\left(\lambda v.vvv\right)x\right)\quad \left(\lambda w.ww\right)\left(uu\right)\quad :\ \left(br,\ xx =_{BIS} vvv\right),\ \left(a1\right);\ \left(br_{\,v-u}\right)$$

$$yyy\left(\left(\lambda v.vvv\right)x\right) \qquad\qquad\qquad :\ \left(br\right),\ \left(a2\right)\ :\ t_1$$

$$yyy\left(xxx\right) =_{BIS} uu\left(uu\right) \qquad :\ \left(br_{\,v-x}\right);\ \left(a1\right),\ \left(br_{\,w-u}\right)$$

$$:\ t_1 =_{BIS} t_2,\ .$$

3.2. Palindromic languages

Even palindromes

$Palin^{rev}_{repl} = \left\{\left(w\, w^{rev}_{iter}\right) \middle| w \in Morph,\ w^{rev} =_{MG} w\right\}$

$Palin^{rev}_{accr} = \left\{\left(w\, w^{rev}_{accr}\right) \middle| w \in Morph,\ w1 \in Palin,\ w1 =_{MG} w2\right\}$

$Palin_{accr} = \left\{\left(ww^{accr}\right) \middle| w \in Morph,\ w1 \in Palin,\ w^{accr} = accr\left(w1\right)\right\}$

Odd palindromes

1. $z \neq tl\left(w1\right) \vee hd\left(w2\right)$

2. $z \notin \{w1,\ w2\}$

$Palin_{rev} = \left\{\left(wzw^{rev}\right) \middle| w,\ z \in Morph,\ z \neq w,\ \left|z\right| \geq 1\right\}$

$Palin_{iter} =$
$$\left\{\left(wzw^{iter}\right) \middle| w \in Morph,\ w1 \in Palin,\ w1 =_{MG} w2,\ z \neq w,\ \left|z\right| \geq 1\right\}$$

$Palin_{accr} = \left\{\left(wzw^{accr}\right) \middle| w \in Morph,\ w1 \in Palin,\right.$

$$w^{accr} = accr\left(w1\right), z \neq w, \left|z\right| \geq 1\}$$

Formula

$$kconcat_{palin}\left(w\,w\right) = \begin{pmatrix} rpl\left(w1\right) = \left[w1w1\right] \in palin \; iff \; w1 \in palin \\ inv\left(w1\right) = \left[w1w1\,^{rev}\right] \in palin \\ accr\left(w1\right) = \left[w1w1\,^{accr}\right] \end{pmatrix}$$

Procedures

Search / selection

Tcontexture $\left(n\right) \Rightarrow$

   List.filter ispalindrome "Tcontexture $\left(n\right)$", ispalindrome : definition.

Construction

Start $\left(morphogram\,w1\right) \Rightarrow$

test $\left(w1\right) \in palin? = \left\{\begin{matrix} yes \\ no \end{matrix}\right\} \Rightarrow$

    composition $\left(w1, w'1\right) \Rightarrow$ test $\left(w1w'1\right) \in palin? = \left\{\begin{matrix} yes \\ no \end{matrix}\right.$

w1 $\notin$ palin $\Rightarrow$ {$ww_{iter}^{rev}, ww_{accr}^{rev}$ }:

- ispalindrome [1,2,2,2,1,2,2,2]; ww_{iter}^{rpl}, w$\neq$palin ??

val it = **false** : bool

- ispalindrome[1,2,2,2,2,2,2,1]; ww_{iter}^{rev}: symmetric palindrome

val it = true : bool

- ispalindrome [1,2,2,2,1,1,1,2]; ww_{accr}^{rev}

val it = true : bool

- ispalindrome[1,2,2,2, 1,1,1,3]; ww_{accr}^{rev} ?? test=no

val it = **false** : bool

- ispalindrome[1,2,2,2,2,2,2,3]; ww_{accr}^{rev}

val it = true : bool

- ispalindrome[1,2,2,2, 3,3,3,2]; ww_{accr}^{rev} ?? test=no

val it = **false** : bool

- ispalindrome[1,2,2,2,3,3,3,1]; ww_{accr}^{rev}

val it = true : bool

- ispalindrome[1,2,2,2,3,3,3,4]; ww_{accr}^{rev}

val it = true : bool

AG([1,2,2,2])+1 = 4: {1,2,3,4}

iter : [1,2] $\rightarrow$ [1,2]

$accr_1$: [1,2] $\rightarrow$ [2,1],

$accr_2$: [1,2] $\rightarrow$ [1,3], ?

   3

   4

   5

$accr_3$: [1,2] → [3,1],

$accr_4$: [1,2] → [3,2], ?

$accr_5$: [1,2] → [3,4].

w1∈ palin ⇒ {ww_{iter}^{rpl}, ww_{accr}^{rpl} }:

- ispalindrome[1,2,2,1,1,2,2,1]; ww_{iter}^{rpl}, symmetric palindrome

val it = true : bool

- ispalindrome [1,2,2,1,2,1,1,2]; ww_{accr}^{rpl}

val it = true : bool

- ispalindrome[1,2,2,1,1,3,3,1]; ww_{accr}^{rpl}

val it = true : bool

- ispalindrome [1,2,2,1, 2,3,3,2]; ww_{accr}^{rpl} ?

val it = **false** : bool

- ispalindrome[1,2,2,1, 3,1,1,3]; ww_{accr}^{rpl} ?

val it = **false** : bool

- ispalindrome [1,2,2,1,3,2,2,3]; ww_{accr}^{rpl}

val it = true : bool

- ispalindrome [1,2,2,1,3,4,4,3]; ww_{accr}^{rpl}

val it = true : bool

- kconcat[1,2,2,1][1,2,2,1];

val it =

[[1,2,2,1,**1,2,2,1**],

[1,2,2,1,**2,1,1,2**],

[1,2,2,1,**1,3,3,1**],

[1,2,2,1,3,1,1,3], ??

[1,2,2,1,2,3,3,2], ??

[1,2,2,1,**3,2,2,3**],

[1,2,2,1,**3,4,4,3**]] : int list list

In the *additive* setting of iteration, kconcat[1,2,2,1][1,2,2,1], the direct repetition (replication) is guaranteed by the unit element of the 'addition', i.e. by the definition of the palindromic construct of replication. Because the concatenation kconcat[1,2,2,1][1,2,2,1] is accretive, not all concatenations are also palindromic. Also the accretion of the palindrome is preserving locally its palindromic structure, the concatenation of palindromes is not generally palindromic.

- kmul[1,2,2,1][1,2,2,1];

 val it =

  [[1,2,2,1,2,1,1,2,2,1,1,2,1,2,2,1],[1,2,2,1,3,1,1,3,3,1,1,3,1,2,2,1],

  [1,2,2,1,2,3,3,2,2,3,3,2,1,2,2,1],[1,2,2,1,3,4,4,3,3,4,4,3,1,2,2,1]]

  : int list list

ispalindrome?

 [[1,2,2,1,**2,1,1,2**,2,1,1,2,**1,2,2,1**],[1,2,2,1,**3,1,1,3**,3,1,1,3,1,2,2,1],

[1,2,2,1,**2,3,3,2**,2,3,3,2,1,2,2,1],[1,2,2,1,**3,4,4,3**,3,4,4,3,1,2,2,1]]
: int list list

In the *multiplicative* setting of iteration, kmul[1,2,2,1][1,2,2,1], the direct repetition (replication) is guaranteed by the unit element of the 'multiplication'. This replication is located in the context of the complexity of the multiplicand. Hence, the replication is embedded in the general concept of iteration by multiplication.

- ispalindrome[1,2,3,4,1,2,3,4];
val it = true : bool

- ispalindrome[1,1,2,1,1,2,1,1]; ww_{iter}^{rev}
val it = true : bool
- ispalindrome [1,1,2,1,2,3,2,2]; ww_{accr}^{rev} ??
val it = **false** : bool
- ispalindrome[1,1,2,1,3,4,3,3]; ww_{accr}^{rev}
val it = true : bool
- ispalindrome[1,1,2,1,1,3,1,1];
val it = true : bool

- ispalindrome [1,1,1,2,2,1,1, 1,1,2,2,1,1,1]; ww_{iter}^{rev}
val it = true : bool
- ispalindrome [1,1,1,2,2,1,1,3,3,4,4,3,3,3]; ww_{accr}^{rev}
val it = true : bool

Iterative replication of the palindrome [1,1,1,2,2,1,1,3,3,4,4,3,3,3]:
- ispalindrome
[1,1,1,2,2,1,1,3,3,4,4,3,3,3,
1,1,1,2,2,1,1,3,3,4,4,3,3,3,
1,1,1,2,2,1,1,3,3,4,4,3,3,3,
1,1,1,2,2,1,1,3,3,4,4,3,3,3,
1,1,1,2,2,1,1,3,3,4,4,3,3,3];
val it = true : bool

Tim Budd's symmetric palindromes in C++
bool palindrome_type1 (string & aString)
    // *test aString is a type 1 palindrome*
{
    string temp; // *declare temporary*
    temp = aString; // *duplicate argument*
    reverse (temp.begin(), temp.end()); // *reverse*
    return temp == aString; // *test for equality*
}

Three step process:
• Duplicate string (uses assignment)
• Reverse duplicate (uses generic algorithm)

• Tests for equality (uses relational operator)

Tim Budd, Data Structures in C++, Chapter 7
http://www.eecis.udel.edu/~saunders/courses/220/04s/buddslides/ch07sli
de.pdf

3.3. Bifunctorial dissemination

Classical or formalistic modeling of the formula follows naturally the *linear* composition of its parts. The composition mode is identity that is enforcing a linear order in the domain of the identity system.

The different modi of similarity and especially the mode of metamorphosis are not anymore organized in a linear order. Because of their 'discontexturality', their order is ruled by mediation and formalized by the extended concept of bifunctoriality.

All three levels of the formula correspond to a different process of reflection. The presentation as a bifunctorial complexion of the formula is therefore the result of a *reflectional* or *praxeological* thematization of the different activities that defines the whole of the formula. In this sense it differs conceptually from a mathematical and formalistic approach of thematization (thinking).

Praxeological analysis
Contexture Cont1 contains the formulation of path1 with W1, W2 and X1. Contexture Cont2 contains the formulation of path2 with W4, W5 and X3. Contexture Cont3 contains the comparison of Cont1 and Cont2 with X1 and X3.

Bifunctoriality for the identity example

Bifunctoriality for similarity type $\begin{bmatrix} 1, 1, 1, 1, 1 \end{bmatrix}$

$$
\begin{bmatrix} W1_1 & - & X1_3 \\ W1_1 & W1_2 & - \\ - & W1_2 & X1_3 \end{bmatrix} :
$$

$$
\left(\begin{pmatrix} \left(W1_1 \circ^{1.0} {}^{.0} W 1_1 \right) \\ \amalg_{1.2} {}^{.0} \\ \left(W1_2 \circ^{0.2} {}^{.0} W1_2 \right) \\ \amalg_{1.2} {}^{.3} \\ \left(X1_3 \circ^{0.0} {}^{.3} X1_3 \right) \end{pmatrix} \right) = \left(\begin{pmatrix} W1_1 \\ \amalg_{1.2} {}^{.0} \\ W1_2 \\ \amalg_{1.2} {}^{.3} \\ X1_3 \end{pmatrix} \right) \circ_{1.2} {}_{.3} \left(\begin{pmatrix} W1_1 \\ \amalg_{1.2} {}^{.0} \\ W1_2 \\ \amalg_{1.2} {}^{.3} \\ X1_3 \end{pmatrix} \right)
$$

Collapses into transitivity :

$$
\left(W1_1 \circ^{1.0} {}^{.0} W 1_1 \right) = \left(W1_1 \right) \circ^{1.0} {}^{.0} \left(W 1_1 \right)
$$

$$
X \equiv WW \equiv \lambda x . F \left(xx \right) W \equiv F \left(WW \right) \equiv FX.
$$

Bifunctoriality for example 1:

Bifunctoriality for similarity type $\begin{bmatrix} 1, 1, 2, 4, 1, 3 \end{bmatrix}$

$$
\begin{bmatrix} W2_1 & - & X3_3 \\ W1_1 & W3_2 & - \\ - & W1_2 & X1_3 \end{bmatrix} :
$$

$$
\left(\begin{pmatrix} \left(W1_1 \circ^{1.0} {}^{.0} W 2_1 \right) \\ \amalg_{1.2} {}^{.0} \\ \left(W1_2 \circ^{0.2} {}^{.0} W3_2 \right) \\ \amalg_{1.2} {}^{.3} \\ \left(X1_3 \circ^{0.0} {}^{.3} X3_3 \right) \end{pmatrix} \right) = \left(\begin{pmatrix} W1_1 \\ \amalg_{1.2} {}^{.0} \\ W1_2 \\ \amalg_{1.2} {}^{.3} \\ X1_3 \end{pmatrix} \right) \circ_{1.2} {}_{.3} \left(\begin{pmatrix} W2_1 \\ \amalg_{1.2} {}^{.0} \\ W3_2 \\ \amalg_{1.2} {}^{.3} \\ X3_3 \end{pmatrix} \right)
$$

Bifunctoriality for example 2:

Bifunctoriality for similarity type $\begin{bmatrix} 1, & 1, & 2, & 4, & 4, & 5 \end{bmatrix}$

$$
\begin{bmatrix}
W2_1 & - & X3_3 \\
W1_1 & W5_2 & - \\
- & W4_2 & X1_3
\end{bmatrix} :
$$

$$
\left(
\begin{pmatrix}
\left(W1_1 \circ^{1.0} {}^{.0} W2_1 \right) \\
\amalg_{1.2}\ ^{.0} \\
\left(W4_2 \circ^{0.2} {}^{.0} W5_2 \right) \\
\\
\amalg_{1.2}\ ^{.3} \\
\left(X1_3 \circ^{0.0} {}^{.3} X3_3 \right)
\end{pmatrix}
\right)
=
\left(
\begin{pmatrix}
W1_1 \\
\amalg_{1.2}\ ^{.0} \\
W4_2 \\
\amalg_{1.2}\ ^{.3} \\
X1_3
\end{pmatrix}
\right)
\circ_{1.2}\ ^{.3}
\left(
\begin{pmatrix}
W2_1 \\
\amalg_{1.2}\ ^{.0} \\
W5_2 \\
\amalg_{1.2}\ ^{.3} \\
X3_3
\end{pmatrix}
\right)
$$

Discontexturality

From a polycontextural point of view, complexity, i.e. polyconteturality is first. Simplicity is a late product of simplification. Domains, contextures or fields are at first considered as discontextural. There is at first no common ground or umbrella to collect the differences into one family and home of sameness or identity.

Full discontexturality without any mediation is the other extreme. To develop and understand a living tissue, different interactions of mediation and separation have to be understood.

Hence, the inverse strategy has to be realized: Not from the simplest scenario to the complexion by deconstructive 'reverse engineering' but from the complex constellation to the reduced cases of simplification.

Because scientific tradition is not offering a complex start, an establishment of complexity has first to start with the approved approaches of scientific simplicity. This strategy might be called "deconstructive re-engineering".

Thereafter, the journey has to elaborate a *double* move from type-[1,1,1,1,1,1] to type[1,2,3,4,5,6] with different length and back from type[1,2,3,4,5,6] to type[1,1,1,1,1,1]:

   Identity: type[1,1,1,1,1,1]
   The most obvious use of "W" is the use in the mode of *identity*:
   Thus, for all i, j, Wi, Wj: Wi $\equiv$ Wj, with i,j$\in$\{1, …,6\},
    and length(W_i) = length(W_j)

i   j)

i

to

Metamorphosis: type[1,2,3,4,5,6]

The most obvious use of "WW" is the use in the mode of *metamorphosis*:

Thus, for all i, j, Wi, Wj: Wi ≢ Wj, with i,j∈{1, ...,6}.

and length(W_i)≠ length(W_j)

and back from *Metamorphosis*: type[1,2,3,4,5,6] to *Identity*: type-[1,1,1,1,1,1].

3.4. Combinatory logical modeling

3.4.1. "Axiomatizing the Quote "

A similar game to the Lambda Calculus is well prepared with the even more direct formalization of Combinatory Logic.

"Specifically, we consider various properties that the *quote* ⌜·⌝ must satisfy as a function from Λ to Λ. The most important of these is the existence of a definable left inverse: a term E, called the *evaluator* for ⌜·⌝, that satisfies E⌜M⌝ = M for all M ∈ Λ. Usually the *quote* ⌜M⌝ encodes the syntax of a given term, and the *evaluator* proceeds by analyzing the syntax and reifying all constructors by their actual meaning in the calculus.

"A coding ⌜·⌝ is a map from C into itself. A term ⌜M⌝ is then called *the quote of M*. Since the primary use of coding consists of manipulating the syntax of terms, most of the properties that we investigate will concern existence of combinators relating the structure of a term to that of its quote. Among these, most attention is given to the *Constructor* and *Destructor* axioms. Roughly, the former allows one to obtain the quote of a term from the quotes of its subterms. The latter is dual: it breaks up the term into its subterms (with respect to the quote)."

"Definition 4. (Coding Axioms) Let ⌜·⌝ : C → C . We say ⌜·⌝ satisfies axiom X from among those below if there exists a combinator X with the stated property.

CON (constructor):

$A : A \lceil M \rceil \lceil N \rceil = \lceil MN \rceil$

$B : B \lceil M \rceil = \lceil \lceil M \rceil \rceil$

E (evaluator) :

$E \lceil M \rceil = M$

Andrew Polonsky, Axiomatizing the Quote
http://subs.emis.de/LIPIcs/volltexte/2011/3249/pdf/36_.pdf
ftp://ftp.cs.kun.nl/pub/CompMath.Found/LC1.pdf, ftp://ftp.cs.kun.nl/pub/CompMath.Found/LC3.pdf

The paper *"Gödel Games: Cloning Gödel's Proofs"* was not considered with *internal* structure of the formulas involved in the process of quotation and interpretation (evaluation).

The difference to this study lies in the morphogrammatic interpretation of the "objects" as morphic palindromes and not (yet) in any syntactic differences of composed formulas like it is presented by the paper *"Axiomatizing the Quote"* of Andrew Polonsky.

Hence, the main constituents of the construction are just the operation of *quotation* and *evaluation*: $\ulcorner \cdot \urcorner : C \to C$ and $E\ulcorner M \urcorner = M$.

The quotation operator $\ulcorner \cdot \urcorner$ shall be replaced by Q, and the evaluator E by the interpreter I, hence "$E\ulcorner M \urcorner = M$" becomes I(Q(M))= M. And "$B\ulcorner M \urcorner = \ulcorner\ulcorner M \urcorner\urcorner$" is replaced by B(Q(M)) = Q(Q(M)).

Of special interest for the highly reduced approach is also A: A(Q(M)Q(N)) = Q(MN), where the retro-grade concatenation of MN enters the game.

3.4.2. Polycontextural modeling

Polycontexturality

$$Q^{(m,n)} : C^{(m,n)} \longrightarrow C^{(m,n)}$$

$$C^{(m,n)} = \begin{pmatrix} C^{1,1} & - & C^{(m,1)} \\ - & (i, j) & - \\ - & - & - \\ C^{(m,1)} & - & C^{(m,n)} \end{pmatrix}$$

grid :

$$I_{act} : G^{i.j} \to G^{i+1.j}$$
$$I_{refl} : G^{i.j} \to G^{i-1.j}$$
$$Q_{act} : G^{i.j} \to G^{i.j+1}$$
$$Q_{refl} : G^{i.j} \to G^{i.j-1}$$

3.4.3. Interpaly of quotation and interpretation

negations :

$$\forall\, i, j \in s(m),\ \forall^{i.j}\, s : I\left(Q_{refl}\left(\neg^{i.j}\, s\right)\right) = \neg^{i-1.j}\left(I\, s\right)$$

$$\forall\, i, j \in s(m),\ \forall^{i.j}\, s : I\left(Q_{act}\left(\neg^{i.j}\, s\right)\right) = \neg^{i.j+1}\left(I\, s\right)$$

junctions + transjunctions :

$$\forall \; i, j \in s\!\left(m\right), \; \forall^{i.j} s, \; t : I\!\left(Q_{refl}\!\left(s \otimes^{i.j} t\right)\right) = I\, s \otimes^{i-1.j} I\, t$$

$$\forall \; i, j \in s\!\left(m\right), \; \forall^{i.j} s, \; t : I\!\left(Q_{act}\!\left(s \otimes^{i.j} t\right)\right) = I\, s \otimes^{i.j+1} I\, t$$

interpreter :

$$\forall \; i, j \in s\!\left(m\right) : I_{refl}\!\left(X_{i.j}\right) = X_{i.j-1}$$

$$\forall \; i, j \in s\!\left(m\right) : I_{act}\!\left(X_{i.j}\right) = X_{i-1.j}$$

$$m \in N, \; I = \left\{I_{refl}, \; I_{act}\right\}, \; \bigotimes = \left\{\wedge, \vee, \rightarrow, <>, \; \triangleleft, \triangleright\right\}$$

reflectional :

$$\forall \; i, j \in s\!\left(m\right) : I_{refl}\!\left(X_{i.j}\right) = X_{i.j-1}$$

$$\forall \; i, j \in s\!\left(m\right) : Q_{refl}\!\left(X_{i.j}\right) = X_{i.j+1}$$

interactional :

$$\forall \; i, j \in s\!\left(m\right) : I_{act}\!\left(X_{i.j}\right) = X_{i-1.j}$$

$$\forall \; i, j \in s\!\left(m\right) : Q_{act}\!\left(X_{i.j}\right) = X_{i+1.j}$$

classic :

$$\forall \; i, j \in s\!\left(m\right) : i = j = 0 : I^0 \equiv I, \; Q^0 \equiv Q.$$

4. General approach

4.1. Modi of morphic self-reference

The general model for morphic self-reference, formalized by normation/diagonalization, gets a specification by the categories of

> replication,
> inversion,
> accretion

ruled by morphic palindromicity.

Morphic palindromicity holds for all kinds of kenogrammatic scriptures. Morphic scriptures are the scriptures of trito-, deutero- and proto-structure of kenogrammatics.

Further *graphematical* scriptures are, e.g., the *indicational* (G. Spencer-Brown) and the *Mersenne* differentiational scripture as sketched in previous papers.

N ∈ morphic scripture
N"N" : (replication, inversion, accretion) modulo palindromicity (ℙ𝕒𝕝)

$$N\,"N\," = \begin{pmatrix} \textbf{quotation modi} \\ \text{replicative} \\ \text{inversive} \\ \text{accretive} \end{pmatrix} \in \mathfrak{Pal}$$

The classical version of self-referentiality is realized as an iterative repetition inscribed in identitive semiotic (symbolic) formal systems (Smullyan).

4.2. Self-reference in other graphematical systems

4.2.1. Indicational (Spencer-Brown) self-reference

Indicational writing system:

re-entry as an iteration of the same in the mode of identity.

4.2.2. Differentiational (Mersenne) self-reference

5. Appendix

kconcat [1,2,2,3][1,2,2,3]
- length(kconcat [1,2,2,3][1,2,2,3]);
val it = 34 : int

*Morphogram*s

[[1,2,2,3,1,2,2,3],[1,**2,2,3,1,3,3,2**],[1,2,2,3,2,1,1,3],[1,2,2,3,2,3,3,1],
[1,2,2,3,3,1,1,2],[1,2,2,3,3,2,2,1],[1,2,2,3,1,2,2,4],[1,2,2,3,1,4,4,2],
[1,2,2,3,2,1,1,4],[1,2,2,3,2,4,4,1],[1,2,2,3,4,1,1,2],[1,2,2,3,4,2,2,1],
[1,2,2,3,1,3,3,4],[1,2,2,3,1,4,4,3],[1,2,2,3,3,1,1,4],[1,2,2,3,3,4,4,1],
[1,2,2,3,4,1,1,3],[1,2,2,3,4,3,3,1],[1,2,2,3,1,4,4,5],[1,2,2,3,4,1,1,5],
[1,2,2,3,4,5,5,1],[1,2,2,3,2,3,3,4],[1,2,2,3,2,4,4,3],[1,2,2,3,3,2,2,4],
[1,2,2,3,3,4,4,2],[1,2,2,3,4,2,2,3],[1,2,2,3,4,3,3,2],[1,2,2,3,2,4,4,5],
[1,2,2,3,4,2,2,5],[1,2,2,3,4,5,5,2],[1,2,2,3,3,4,4,5],[1,2,2,3,4,3,3,5],
[1,2,2,3,4,5,5,3],[1,2,2,3,4,5,5,6]]

Non-palindrome, one from 20
- ispalindrome [1,2,2,3,1,3,3,2];
val it = false : bool

palindromes:
[1,2,2,3,**1,2,2,3**], [1,2,2,3,**3,2,2,1**]
[1,2,2,3,**1,4,4,3**], [1,2,2,3,**3,4,4,1**]

Palindromes:
List.filter ispalindrome "kconcat [1,2,2,3][1,2,2,3]":

val it =
  [[1,2,2,3,1,2,2,3],[1,2,2,3,2,3,3,1],[1,2,2,3,3,1,1,2],,
   [1,2,2,3,4,1,1,2],[1,2,2,3,4,2,2,1],[1,2,2,3,1,4,4,3],[1,2,2,3,3,4,4,1],
   [1,2,2,3,4,5,5,1],[1,2,2,3,2,3,3,4],[1,2,2,3,3,2,2,4],[1,2,2,3,4,2,2,5],
   [1,2,2,3,3,4,4,5],[1,2,2,3,4,5,5,6]] : int list list
- length it;
val it = 14 : int

Symmetric palindrome
val it = [[1,2,2,3,3,2,2,1]] : int list list

reverse tail palindromes:
[1,2,2,3,**1,2,2,3**], [1,2,2,3,**3,2,2,1**]
[1,2,2,3,**1,4,4,3**], [1,2,2,3,**3,4,4,1**]

Modi for palindromes with head=[1,2,2,3] out of kconcat [1,2,2,3][1,2,2,3]:
replication: [1,2,2,3,1,2,2,3],   : palin1+palin1
inversion: [1,2,2,3,3,2,2,1],   : palin1+ rev(palin1)
accretion: [1,2,2,3,2,3,3,1],[1,2,2,3,3,1,1,2], : palin1+accr(palin1)
               [1,2,2,3,1,4,4,3],[1,2,2,3,3,4,4,1]
               [1,2,2,3,4,1,1,2],[1,2,2,3,4,2,2,1], [1,2,2,3,2,3,3,4],
               [1,2,2,3,4,5,5,1],[1,2,2,3,3,2,2,4],[1,2,2,3,4,2,2,5],
               [1,2,2,3,4,5,5,6],[1,2,2,3,3,4,4,5].

kmul[1,2,2,3][1,2,2,3];
- length it;
val it = 588 : int

List.filter ispalindrome "kmul[1,2,2,3][1,2,2,3]":
val it =
  [[1,2,2,3,2,3,3,1,2,3,3,1,3,1,1,2],[1,2,2,3,3,1,1,2,3,1,1,2,2,3,3,1],
   [1,2,2,3,2,1,1,4,2,1,1,4,3,4,4,1],[1,2,2,3,2,1,1,4,2,1,1,4,5,4,4,1],
   [1,2,2,3,2,4,4,1,2,4,4,1,3,1,1,2],[1,2,2,3,2,4,4,1,2,4,4,1,5,1,1,2],
   [1,2,2,3,4,1,1,2,4,1,1,2,3,4,4,1],[1,2,2,3,4,1,1,2,4,1,1,2,5,4,4,1],
   [1,2,2,3,3,1,1,4,3,1,1,4,4,5,5,1],[1,2,2,3,4,3,3,1,4,3,3,1,3,5,5,4],
   [1,2,2,3,4,1,1,5,4,1,1,5,2,3,3,1],[1,2,2,3,4,1,1,5,4,1,1,5,3,6,6,1],
   [1,2,2,3,4,1,1,5,4,1,1,5,6,7,7,1],[1,2,2,3,4,5,5,1,4,5,5,1,2,3,3,4],
   [1,2,2,3,4,5,5,1,4,5,5,1,3,6,6,4],[1,2,2,3,4,5,5,1,4,5,5,1,6,7,7,4],
   [1,2,2,3,2,3,3,4,2,3,3,4,3,4,4,1],[1,2,2,3,2,3,3,4,2,3,3,4,3,4,4,5],
   [1,2,2,3,3,4,4,2,3,4,4,2,2,3,3,1],[1,2,2,3,3,4,4,2,3,4,4,2,2,3,3,5],
   [1,2,2,3,4,3,3,2,4,3,3,2,3,4,4,1],[1,2,2,3,4,3,3,2,4,3,3,2,3,4,4,5],
   [1,2,2,3,2,4,4,5,2,4,4,5,3,5,5,1],[1,2,2,3,2,4,4,5,2,4,4,5,6,5,5,1],
   [1,2,2,3,2,4,4,5,2,4,4,5,3,5,5,6],[1,2,2,3,2,4,4,5,2,4,4,5,6,5,5,7],
   [1,2,2,3,4,5,5,2,4,5,5,2,3,4,4,1],[1,2,2,3,4,5,5,2,4,5,5,2,6,4,4,1],
   [1,2,2,3,4,5,5,2,4,5,5,2,3,4,4,6],[1,2,2,3,4,5,5,2,4,5,5,2,6,4,4,7],
   [1,2,2,3,3,4,4,5,3,4,4,5,5,1,1,2],[1,2,2,3,3,4,4,5,3,4,4,5,5,6,6,1],
   [1,2,2,3,3,4,4,5,3,4,4,5,5,6,6,7],[1,2,2,3,4,3,3,5,4,3,3,5,3,1,1,2],
   [1,2,2,3,4,3,3,5,4,3,3,5,3,6,6,1],[1,2,2,3,4,3,3,5,4,3,3,5,3,6,6,7],
   [1,2,2,3,4,5,5,6,4,5,5,6,2,3,3,1],[1,2,2,3,4,5,5,6,4,5,5,6,3,1,1,2],

  [1,2,2,3,4,5,5,6,4,5,5,6,7,1,1,2],[1,2,2,3,4,5,5,6,4,5,5,6,3,7,7,1],
  [1,2,2,3,4,5,5,6,4,5,5,6,7,8,8,1],[1,2,2,3,4,5,5,6,4,5,5,6,2,3,3,7],
  [1,2,2,3,4,5,5,6,4,5,5,6,3,7,7,8],[1,2,2,3,4,5,5,6,4,5,5,6,7,8,8,9]] : int list list
- length it;
val it = 44 : int

Decomposition example
  [1,2,2,3, 2,3,3,1, 2,3,3,1, 3,1,1,2]:
  [1,2,2,3,2,3,3,1],[1,2,2,3,2,3,3,1],[1,2,2,3,3,1,1,2] in Tcontexture 8

 Matrix notation
    (1)   (2)   (3)
 [1,2,2,3] [2,3,3,1] [3,1,1,2]
         [2,3,3,1]

- kmul [1,2][1,2];
val it = [[1,2,2,1],[1,2,3,1],[1,2,2,3],[1,2,3,4]] : int list list
- List.filter ispalindrome [[1,2,2,1],[1,2,3,1],[1,2,2,3],[1,2,3,4]];
val it = [[1,2,2,1],[1,2,3,1],[1,2,2,3],[1,2,3,4]] : int list list

Modi for head=[1,2] for palindromes out of "kmul[1,2][1,2]"
repetition: –
inversion: [1,2,**2,1**],
accretion: [1,2,3,1],[1,2,2,3],[1,2,3,4].

- kconcat [1,2][1,2];
val it =
  [[1,2,1,2],[1,2,2,1],[1,2,1,3],[1,2,3,1],[1,2,2,3],[1,2,3,2],[1,2,3,4]]: int list list
ispalindrome:
val it = [[1,2,1,2],[1,2,2,1],[1,2,3,1],[1,2,2,3],[1,2,3,4]] : int list list
repetition: [1,2,1,2],
inversion: [1,2,2,1],
accretion: [1,2,3,1],[1,2,2,3],[1,2,3,4].